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MATH 101
HOMEWORK 3 SOLUTIONS

1. Find $f'(x)$ where $f(x) = \tan(\cos^2(x + \sec x) - x^3 + 1)$.

Solution:
$f'(x) = \sec^2\left(\cos^2(x+\sec x) - x^3 + 1\right)\left(2\cos(x+\sec x)(-\sin(x+\sec x))(1+\sec x\tan x) - 3x^2\right).$

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2. Find $f'(x)$ where $f(u) = \frac{1}{u}$, $u(t) = t^2 + \cos t$, $t(x) = (1+3x)^9$.

Solution:
$f(x) = \frac{1}{(1+3x)^{18} + \cos[(1+3x)^9]}.$

$f'(x) = -\frac{18(1+3x)^{17}(3) - \sin[(1+3x)^9](9(1+3x)^8(3))}{[(1+3x)^{18} + \cos[(1+3x)^9]^2}.$

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3. Find $f'(x)$ where $f(x) = \left(\sec(x^2 + \tan x) + x^5\right)^2 \left(x^2 + x + 1\right)^3$.

Solution:

$$\begin{aligned} f'(x) &= 2\left(\sec(x^2+\tan x) + x^5\right)\Big[\sec(x^2+\tan x)\tan(x^2+\tan x)\cdot(2x+\sec^2 x) \\ &\quad +5x^4\Big](x^2+x+1)^3 \\ &\quad + \left(\sec(x^2+\tan x)+x^5\right)^2[3(x^2+x+1)^2(2x+1)]. \end{aligned}$$

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4. $x(t) = t^2 + \cos t$, $y(t) = t + \sin t$. Find $\frac{dy}{dx}$.

Solution:

$\frac{dy}{dx} = \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})} = \frac{1+\cos t}{2t - \sin t}.$

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5. $x(t) = t^2 + t - \tan t$, $y(t) = t^3 + \cot t$. Find $\frac{dy}{dx}$.

Solution:

$\frac{dy}{dx} = \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})} = \frac{3t^2 - \csc^2 t}{2t + 1 - \sec^2 t}.$

6. $x(t) = 3\cos t$, $y(t) = 5\sin t$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

Solution:

$$\frac{dy}{dx} = \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})} = \frac{5\cos t}{-3\sin t}.$$

$$\frac{d^2y}{dx^2} = \frac{(\frac{(d[dy/dx])}{dt})}{(\frac{dx}{dt})} = \frac{(d[\frac{5\cos t}{-3\sin t}]/dt)}{-3\sin t} = \frac{1}{-3\sin t}\left(\frac{(-5\sin t)(-3\sin t) - (5\cos t)(-3\cos t)}{9\sin^2 t}\right).$$

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7. If $x^2 + 3xy + y^2 = 10$, then find $\dfrac{dy}{dx}$.

Solution:

$2x + 3y + 3xy' + 2yy' = 0$ and $y' = -\dfrac{2x+3y}{3x+2y}$.

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8. If $y\cos x + x\sin y = 8$, then find $\dfrac{dy}{dx}$.

Solution:

$y'\cos x - y\sin x + \sin y + x\cos y \cdot y' = 0$ and $y' = \dfrac{y\sin x - \sin y}{\cos x + x\cos y}$.

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9. If $x^7 + xy + y^7 = 3$, then find $\dfrac{dy}{dx}$ and $\dfrac{dy}{dx}(1,1)$.

Solution:

$7x^6 + y + xy' + 7y^6y' = 0$ and $y' = -\dfrac{7x^6+y}{x+7y^6}$. Putting in $x = 1,\ y = 1$ we get $y'(1,1) = -1$.

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10. If $\tan(xy) + 1 = 0$, then find $\dfrac{dy}{dx}$ and $\dfrac{dy}{dx}(-4, \pi/16)$.

Solution:

$\sec^2(xy)\cdot(y + xy') = 0$, but sec is never zero so we must have $(y + xy') = 0$, from where we find $y' = -\dfrac{y}{x}$. Now putting $x = -4,\ y = \pi/16$ we get $y'(-4, \pi/16) = \pi/64$.

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