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MATH 101 HOMEWORK-4 SOLUTIONS

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Page 278, Exercise 26-a. $P = 2x + 2y = 36$, $y = 18 - x$. When the cylinder is formed $r = x/(2\pi)$ and $h = y$. The volume of the cylinder then becomes $V = \pi r^2 h = (18x^2 - x^3)/(4\pi) = V(x)$. Solving $V'(x) = 3x(12 - x)/(4\pi) = 0$ we get $x = 0$ or $x = 12$. When $x = 0$ we have no cylinder. Since $V''(x) = 3(6 - x)/(2\pi)$ and $V''(12) < 0$ we have a maximum value at $x = 12$, $y = 6$.

Page 278, Exercise 26-b. In this case $V(x) = \pi x^2(18-x)$. Solving $V'(x) = 3\pi x(12-x) = 0$ we get $x = 0$ or $x = 12$. As before $x = 0$ gives no cylinder. Since $V''(x) = 6\pi(6 - x)$ and $V''(12) < 0$, we have a maximum value at $x = 12$, $y = 6$.

Page 278, Exercise 27. Here $h^2 + r^2 = 3$. The volume is $V = (\pi/3)r^2h = (\pi/3)(3 - h^2)h$ for $0 < h < \sqrt{3}$. $V'(h) = \pi(1 - h^2) = 0$ gives $h = 1$. Since $V''(1) < 0$ we have a maximum value at $h = 1$. Then $r = \sqrt{2}$ and $V = (2\pi/3)$.

Page 279, Exercise 32. Let x be the distance from the point on the shoreline nearest Jane's boat to the point where she lands her boat. Then she needs to row $\sqrt{4 + x^2}$ miles at 2 mph and walk $6 - x$ miles at 5 mph. The total amount of time to reach the village is $f(x) = \sqrt{4 + x^2}/2 + (6 - x)/5$ hours, $0 \le x \le 6$. Solving for $f'(x) = 0$ in this domain we find $x = 4/\sqrt{21}$. Checking the end points and this critical value we find $f(0) = 2.2$, $f(4/\sqrt{21}) = 2.12..$, $f(6) = 3.16...$ Hence for the shortest travel time she should land her boat $4/\sqrt{21}$ miles down the shoreline from the point nearest her boat.

Page 279, Exercise 33. Let h denote the height of the beam from the ground when it is leaning on the wall and x denote the distance of the other end of the beam from the 8 feet wall. Then we have $8/x = h/(x+27)$, giving us $h = 8 + \frac{216}{x}$. The length of the beam is then $L(x) = \sqrt{h^2 + (x + 27)^2} = \sqrt{(8 + \frac{216}{x})^2 + (x + 27)^2}$ with $x > 0$. Note that $L(x)$ is minimized when $f(x) = (8 + \frac{216}{x})^2 + (x + 27)^2$ is minimized. $f'(x) = 2(8 + \frac{216}{x})(-\frac{216}{x^2}) + 2(x + 27) = \frac{2}{x^3}(x^4 + 27x^3 - 1728x - 46656) = \frac{2}{x^3}(x - 12)(x + 27)(x^2 + 12x + 144)$. Solving for $f'(x) = 0$ in the domain we find $x = 12$. Since $f(x)$ becomes infinite as x approaches 0 and ∞, and since this is the only critical point, it should give the global minimum value. Hence the shortest possible length for the beam is $L(12) = \sqrt{f(12)} = \sqrt{2197} = 46.87..$ feet.

Page 307, Exercise 44. $A(x) = \frac{1}{2}(2x)(27 - x^2) = 27x - x^3$, $0 \le x \le 3\sqrt{3}$.
$A'(x) = 27 - 3x^2 = 0 \Rightarrow \; x = 3$. $A''(x) = -6x$. $A''(3) < 0 \;\Rightarrow\; A(3) = 54$ is the maximal value.
