

MATH 101 HOMEWORK-5 SOLUTIONS

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Page 387, Exercise 59. $\sqrt{x} + \sqrt{y} = 1$; $y = (1 - \sqrt{x})^2$.

$$\text{Area} = \int_0^1 (1 - \sqrt{x})^2 dx = \int_0^1 (1 - 2\sqrt{x} + x) dx = \left(x - \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 \Big|_0^1 \right) = \frac{1}{6}.$$

Page 387, Exercise 60. $x^3 + \sqrt{y} = 1$; $y = (1 - x^3)^2$.

$$\text{Area} = \int_0^1 (1 - x^3)^2 dx = \int_0^1 (1 - 2x^3 + x^6) dx = \left(x - \frac{1}{2}x^4 + \frac{1}{7}x^7 \Big|_0^1 \right) = \frac{9}{14}.$$

Page 387, Exercise 66. $x^{1/3} + y^{1/3} = 1$; $y = (1 - x^{1/3})^3$.

$$\text{Area} = \int_0^1 (1 - x^{1/3})^3 dx = \int_0^1 (1 - 3x^{1/3} + 3x^{2/3} - x) dx = \left(x - \frac{9}{4}x^{4/3} + \frac{9}{5}x^{5/3} - \frac{1}{2}x^2 \Big|_0^1 \right) = \frac{1}{20}.$$

Page 391, Exercise 19. $f(x) = \int_{1/x}^x \frac{1}{t} dt$;

$$f'(x) = \frac{1}{x}(x)' - \frac{1}{(1/x)} \left(\frac{1}{x} \right)' = \frac{2}{x}.$$

Page 391, Exercise 20. $f(x) = \int_{\cos x}^{\sin x} \frac{1}{1-t^2} dt$;

$$f'(x) = \frac{1}{1 - \sin^2 x} (\sin x)' - \frac{1}{1 - \cos^2 x} (\cos x)' = \frac{1}{\cos x} + \frac{1}{\sin x}.$$
