



Quiz # 1
Math 101-Section 09 Calculus I
17 September 2015, Thursday



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YOUR NAME:

In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) We have a function defined as

$$f(x) = \begin{cases} x^2 + cx & \text{if } x \geq 1, \\ x + 2 & \text{if } x < 1. \end{cases}$$

Here c is a constant to be determined below.

- (i) Determine c such that f becomes continuous at $x = 1$ Use this value of c for the following questions.
- (ii) Calculate $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1}$.
- (iii) Calculate $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1}$.
- (iv) Calculate $f'(1)$ if it exists.

: Grading is 25 points for each correct question.

Answer:

(i) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 + cx) = 1 + c$. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x + 2) = 3$.

For f to be continuous at $x = 1$ these right and left limits must be the same and both must be equal to $f(1)$. This holds when $c = 2$. So set $c = 2$ in the following solutions.

(ii) $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{x + 2 - 3}{x - 1} = 1$.

(iii) $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{x^2 + 2x - 3}{x - 1} = \lim_{x \rightarrow 1^+} \frac{(x - 1)(x + 3)}{x - 1} = \lim_{x \rightarrow 1^+} (x + 3) = 4$.

(iv) From (ii) and (iii) above, we see that $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$ does not exist, since the right and left limits do not agree. Hence $f'(1)$ does not exist.