Bilkent University
Department of Mathematics
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## Quiz \# 4

Math 101-Section 09 Calculus I
13 October 2015, Tuesday

## YOUR NAME:

## In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) We have an ice block in the shape of a right circular cylinder which is melting. We measure that when the height is 5 cm and the radius is 12 cm , the radius is decreasing at the rate of $3 \mathrm{~cm} / \mathrm{sec}$ and the block is melting at the rate of $504 \pi \mathrm{~cm}^{3} / \mathrm{sec}$.
(a) How fast is the height decreasing at that moment?
(b) Write a linearization of the volume of the block at that moment and estimate what its volume will be 1 sec later. Hint: All of this is happening at time $t=0$.

Grading is 50 points for each part.

## Answer:

The volume of the cylinder is given by $V=\pi r^{2} h$. Taking derivative of both sides with respect to time, we get

$$
V^{\prime}=2 \pi r h r^{\prime}+\pi r^{2} h^{\prime} .
$$

Substituting the given data into this equation we have

$$
-504 \pi=-360 \pi+144 \pi h^{\prime} .
$$

Solving for $h^{\prime}$ we find

$$
h^{\prime}=-1 .
$$

This means that the height is decreasing at the rate of $1 \mathrm{~cm} / \mathrm{sec}$ at that time.
The volume at that time is $V_{0}=720 \pi \mathrm{~cm}^{3}$, and $V^{\prime}=-504 \pi \mathrm{~cm}^{3} / \mathrm{sec}$. The linearization is

$$
L(t)=-504 \pi t+720 \pi .
$$

Hence one second later, the volume is estimated to be

$$
L(1)=216 \pi \mathrm{~cm}^{3} .
$$

