



Quiz # 6
Math 101-Section 011 Calculus I
17 November 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

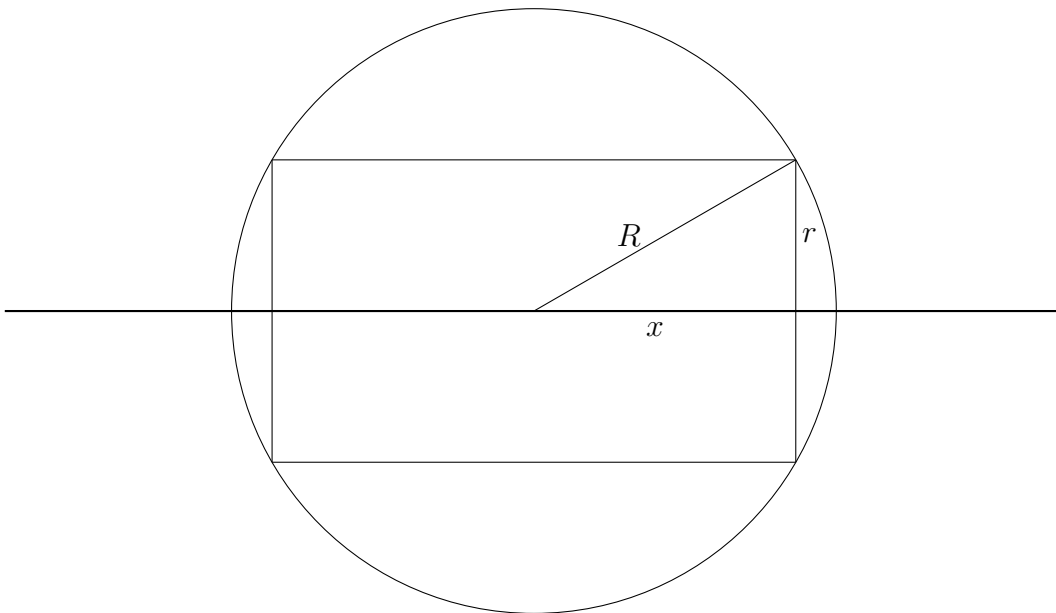
Student ID:

Your Department:

Show your work in detail. Correct answers without justification are never graded.

Q-1) Find the maximum volume for a right circular cylinder inscribed in a sphere of radius R . (10 points)

Answer:



Height of the cylinder is $2x$ and the base radius is r . From the figure we have $r^2 = R^2 - x^2$. Therefore the volume is given by

$$V(x) = 2x \pi r^2 = 2x \pi (R^2 - x^2) = 2\pi (R^2 x - x^3), \quad \text{where } 0 \leq x \leq R.$$

We first find the critical points.

$$V'(x) = 2\pi (R^2 - 3x^2) = 0 \quad \text{gives } x = \frac{R}{\sqrt{3}} \quad \text{in the domain.}$$

We now check the values of $V(x)$ at the critical point and at the end points.

$$V(0) = 0, \quad V\left(\frac{R}{\sqrt{3}}\right) = \frac{4\pi R^3}{3\sqrt{3}}, \quad V(R) = 0.$$

Hence the maximum possible volume of a right circular cylinder inscribed into a sphere of radius R is $\frac{4\pi R^3}{3\sqrt{3}} \approx 2.42R^3$, which is $\frac{1}{\sqrt{3}}$ of the total volume of the sphere.