Quiz # 6
Math 101-Section **01** Calculus I
10 November 2017, Friday
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Solution Key

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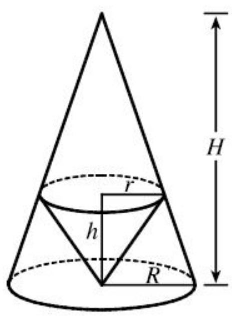
Your Name: ....

Your Student ID: ....

Q-1) A cone with height h is inscribed in a larger cone of height H so that its vertex is at the center of the base of the larger cone. Find the maximum possible volume of the inscribed cone in terms of the volume of the larger cone. (*This is Exercise 43 in your textbook on page 266. I am assuming that you are working on these problems before coming to class!*) 10 points

Solution: Here is the official solution!



By similar triangles, $\dfrac{H}{R} = \dfrac{H-h}{r}$ **(1)**. The volume of the inner cone is $V = \frac{1}{3}\pi r^2 h$, so we'll solve **(1)** for h. $\dfrac{Hr}{R} = H - h \Rightarrow$

$$h = H - \frac{Hr}{R} = \frac{HR - Hr}{R} = \frac{H}{R}(R - r) \quad \textbf{(2)}.$$

Thus, $V(r) = \dfrac{\pi}{3} r^2 \cdot \dfrac{H}{R}(R-r) = \dfrac{\pi H}{3R}(Rr^2 - r^3) \Rightarrow$

$$V'(r) = \frac{\pi H}{3R}(2Rr - 3r^2) = \frac{\pi H}{3R} r(2R - 3r).$$

$V'(r) = 0 \Rightarrow r = 0$ or $2R = 3r \Rightarrow r = \frac{2}{3}R$ and from **(2)**, $h = \dfrac{H}{R}\left(R - \frac{2}{3}R\right) = \dfrac{H}{R}\left(\frac{1}{3}R\right) = \frac{1}{3}H$.

$V'(r)$ changes from positive to negative at $r = \frac{2}{3}R$, so the inner cone has a maximum volume of

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2}{3}R\right)^2\left(\frac{1}{3}H\right) = \frac{4}{27} \cdot \frac{1}{3}\pi R^2 H$, which is approximately 15% of the volume of the larger cone.

So the maximal possible volume for the small cone is $\dfrac{4}{27}$ of the volume of the larger cone.