Quiz # 7
Math 101-Section **01** Calculus I
17 November 2017, Friday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Your Student ID: .....

Q-1) Interpret the following limit as a Riemann sum and find the limit as the value of that definite integral:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n\sqrt{n^2+k^2}}.$$

10 points

Solution:

We have

$$\frac{k}{n\sqrt{n^2+k^2}} = \frac{1}{n}\,\frac{k/n}{\sqrt{1+(k/n)^2}} = f(x_k)\,\Delta x,$$

where we take $[a,b]=[0,1]$, $f(x)=\dfrac{x}{\sqrt{1+x^2}}$, $\Delta x = \dfrac{1}{n}$ and $x_k = \dfrac{k}{n}$.

Note that if we take $F(x)=\sqrt{1+x^2}$, then $F'(x)=f(x)$. Hence we have

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n\sqrt{n^2+k^2}} = \int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx = \left(\sqrt{1+x^2}\Big|_0^1\right) = \sqrt{2}-1 \approx 0.4142.$$