Quiz # 8
Math 101-Section **01** Calculus I
24 November 2017, Friday
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Solution Key

Bilkent University

Your Name: .....

Your Student ID: .....

Q-1)

(a) Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt[3]{(1+2x)^2}}$.

(b) Evaluate $\displaystyle\int \sin^3\theta\cos^2\theta\,d\theta$.

(c) Find y if $\dfrac{dy}{dt} = (1+\sin t)\sqrt{y}$ and $y(0) = 16$.

3+4+3 points

Solution:

(a) Putting $u = 1+2x$ we get

$$\int_0^1 \frac{dx}{\sqrt[3]{(1+2x)^2}} = \frac{1}{2}\int_1^3 u^{-2/3}\,du = \frac{3}{2}\left(u^{1/3}\Big|_1^3\right) = \frac{3}{2}(3^{1/3}-1) \approx 0.66$$

(b) Writing $\sin^3\theta = (1-\cos^2\theta)\sin\theta$ and putting $u = \cos\theta$ we get

$$\int \sin^3\theta\cos^2\theta\,d\theta = \int (u^4-u^2)\,d\theta = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \frac{1}{5}\cos^5\theta - \frac{1}{3}\cos^3\theta + C.$$

(c) Cross-multiplication gives

$$\frac{dy}{\sqrt{y}} = (1+\sin t)\,dt.$$

Integrating both sides we get

$$2\sqrt{y} = t - \cos t + C, \quad \text{or} \quad y = \frac{1}{4}(t-\cos t + C)^2.$$

Putting in $y(0) = 16$ we see that $C = 9$, hence

$$y = \frac{1}{4}(t-\cos t + 9)^2.$$