Bilkent University

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Q-1) Let $f(x) = \dfrac{x^2 - 1}{1 + x^2 + x^4}$, and $g(x) = \dfrac{1 + \cos x}{2 - \sin x}$.

     (i) Calculate $(f \circ g)' \left(\frac{\pi}{2} \right)$.

     (ii) Calculate $(g \circ f)' (1)$.

Answer:

$$
\begin{aligned}
g'(x) &= \frac{(-\sin x)(2 - \sin x) - (1 + \cos x)(-\cos x)}{(2 - \sin x)^2} \\[2mm]
g'(\frac{\pi}{2}) &= -1 \\[2mm]
f'(x) &= \frac{(2x)(1 + x^2 + x^4) - (x^2 - 1)(4x^3 + 2x)}{(x^4 + x^2 + 1)^2} \\[2mm]
g(\frac{\pi}{2}) &= 1 \\[2mm]
f'(1) &= \frac{2}{3} \\[2mm]
(f \circ g)' \left(\frac{\pi}{2}\right) &= f'(g(\frac{\pi}{2}))\, g'(\frac{\pi}{2}) = f'(1)\, g'(\frac{\pi}{2}) = -\frac{2}{3}.
\end{aligned}
$$

$$
\begin{aligned}
f(1) &= 0 \\[2mm]
g'(0) &= \frac{1}{2} \\[2mm]
(g \circ f)' (1) &= g'(f(1))\, f'(1) = g'(0) f'(1) = \frac{1}{3}.
\end{aligned}
$$