Bilkent University

Name: ....

Department: ....

Student ID: ....

Q-1)

(i) The hypothenuse of a right triangle is increasing at the rate of $11/63$ cm/s when it is 5 cm, and at that time one of the legs is 4 cm and is decreasing at the rate of $2/9$ cm/s. Find how fast the other leg is changing at that time.

(ii) Find the absolute min/max of $f(x) = x^4 + 4x^3 + 4x^2$ on $[-1.5, 1]$.

Answer (i) : Using the right triangle theorem, $x^2 + y^2 = z^2$, we find that the other leg is 3 cm at that time. $(3^2 + 4^2 = 5^2)$

Differentiating the above equation with respect to time, and cancelling out the 2, we get

$$x\,x' + y\,y' = z\,z'.$$

Putting in the values,

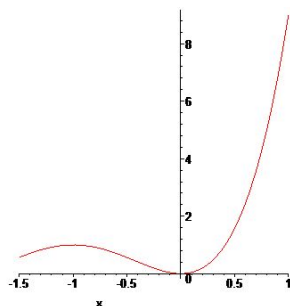$$3x' + (4)\left(-\frac{2}{9}\right) = (5)\left(\frac{11}{63}\right),$$

we find that $x' = \dfrac{37}{63}$ cm/s.

Answer (ii) : $f'(x) = 4x^3 + 12x^2 + 8x = 4x(x+1)(x+2) = 0.$

The critical points in the given domain are 0 and -1. We evaluate f at these critical and end points.

$$f(-3/2) = 9/16 \approx 0.56, \quad f(-1) = 1, \quad f(0) = 0, \quad f(1) = 9.$$

Therefore the absolute min is 0, and the absolute max is 9.

Here is the graph