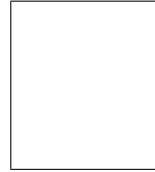




Quiz # 8
 Math 101-Section 06 Calculus I
 5 April, 2018, Thursday
 Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Name:

Department:

Student ID:

Q-1)

(i) Express $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2}{k^3 + n^3}$ as a definite integral; do not evaluate the integral

(ii) Evaluate $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} (7 + \frac{1}{\cos x})^{1/3} dx$.

Answer:

We want the summand to look like $f(x_k)\Delta x_k$ where $x_k = k/n$ and $\Delta x_k = 1/n$.

$$\frac{k^2}{k^3 + n^3} = \frac{k^2 n}{k^3 + n^3} \frac{1}{n} = \frac{(\frac{k}{n})^2}{(\frac{k}{n})^3 + 1} \frac{1}{n} = \frac{x^2}{x^3 + 1} \Delta x.$$

Hence

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2}{k^3 + n^3} = \int_0^1 \frac{x^2}{x^3 + 1} dx.$$

(ii) Let $u = 7 + \sec x$, then $du = \sec x \tan x dx$, and

$$\begin{aligned} \int_{x=0}^{x=\pi/3} \frac{\sin x}{\cos^2 x} (7 + \frac{1}{\cos x})^{1/3} dx &= \int_{x=0}^{x=\pi/3} \sec x \tan x (7 + \sec x)^{1/3} dx \\ &= \int_{x=0}^{x=\pi/3} u^{1/3} du \\ &= \left(\frac{3}{4} u^{4/3} \Big|_{x=0}^{x=\pi/2} \right) \\ &= \left(\frac{3}{4} (7 + \sec x)^{4/3} \Big|_{x=0}^{x=\pi/2} \right) \\ &= \frac{27}{4} 3^{2/3} - 12 \\ &\approx 2.04 \end{aligned}$$