Quiz # 8
Math 101-Section **06** Calculus I
5 April, 2018, Thursday
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Solution Key

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Q-1)

(i) Express $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{k^3+n^3}$ as a definite integral; do not evaluate the integral

(ii) Evaluate $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}(7+\frac{1}{\cos x})^{1/3}\,dx$.

Answer:

We want the summand to look like $f(x_k)\Delta x_k$ where $x_k = k/n$ and $\Delta x_k = 1/n$.

$$\frac{k^2}{k^3+n^3} = \frac{k^2 n}{k^3+n^3}\frac{1}{n} = \frac{\left(\frac{k}{n}\right)^2}{\left(\frac{k}{n}\right)^3+1}\frac{1}{n} = \frac{x^2}{x^3+1}\,\Delta x.$$

Hence

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{k^3+n^3} = \int_0^1 \frac{x^2}{x^3+1}\,dx.$$

(ii) Let $u = 7+\sec x$, then $du = \sec x \tan x\,dx$, and

$$\begin{aligned}
\int_{x=0}^{x=\pi/3} \frac{\sin x}{\cos^2 x}(7+\frac{1}{\cos x})^{1/3}\,dx &= \int_{x=0}^{x=\pi/3} \sec x\tan x(7+\sec x)^{1/3}\,dx \\
&= \int_{x=0}^{x=\pi/3} u^{1/3}\,du \\
&= \left(\frac{3}{4}u^{4/3}\Big|_{x=0}^{x=\pi/2}\right) \\
&= \left(\frac{3}{4}(7+\sec x)^{4/3}\Big|_{x=0}^{x=\pi/2}\right) \\
&= \frac{27}{4}\,3^{2/3} - 12 \\
&\approx 2.04
\end{aligned}$$