Quiz # 9
Math 101-Section **01** Calculus I
13 April, 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

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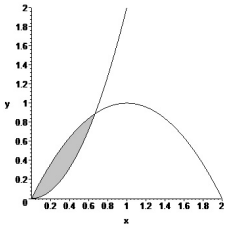
Q-1) Let $k > 0$ be a real number.

(i) Find the area bounded between the curves $y = kx^2$ and $y = 2x - x^2$. This is the shaded area in the figure.

(ii) Revolve the shaded area about the x-axis and find the volume so obtained.



Answer:

(i)

$$\text{Area} = \int_0^{\frac{2}{k+1}} [(2x - x^2) - (kx^2)]\, dx = \left(x^2 - \frac{x^3}{3} - \frac{kx^3}{3} \Big|_0^{\frac{2}{k+1}} \right) = \frac{4}{3(k+1)^2}.$$

(ii)

$$\text{Volume} = \pi \int_0^{\frac{2}{k+1}} [(2x - x^2)^2 - (kx^2)^2]\, dx = \pi \left(\frac{x^5}{5} - x^4 + \frac{4x^3}{3} - \frac{k^2x^5}{5} \Big|_0^{\frac{2}{k+1}} \right) = \frac{16(4k+1)}{15(k+1)^4}\pi.$$