Quiz # 9
Math 101-Section **06** Calculus I
12 April, 2018, Thursday
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Solution Key

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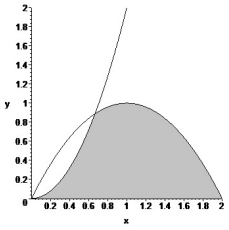
Q-1) Let $k > 0$ be a real number.

(i) Find the area bounded by the curves $y = kx^2$, $y = 2x - x^2$ and $y = 0$. This is the shaded area in the figure.

(ii) Revolve the shaded area about the x-axis and find the volume so obtained.



Answer:

$$\begin{aligned}
\text{Area} &= \int_0^{\frac{2}{k+1}} kx^2\,dx + \int_{\frac{2}{k+1}}^{2} (2x - x^2)\,dx \\
&= \left(\frac{kx^3}{3}\Big|_0^{\frac{2}{k+1}} \right) + \left(x^2 - \frac{x^3}{3}\Big|_{\frac{2}{k+1}}^{2} \right) \\
&= \frac{4k(k+2)}{3(k+1)^2}.
\end{aligned}$$

(ii)

$$\begin{aligned}
\text{Volume} &= \int_0^{\frac{2}{k+1}} (kx^2)^2\,dx + \int_{\frac{2}{k+1}}^{2} (2x - x^2)^2\,dx = \left(\frac{k^2x^5}{5}\Big|_0^{\frac{2}{k+1}} \right) + \left(\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\Big|_{\frac{2}{k+1}}^{2} \right) \\
&= \frac{16}{15}\,\frac{k^2\,(k^2 + 4\,k + 6)}{(k+1)^4}
\end{aligned}$$