Bilkent University

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Q-1)

(i) Find and simplify the derivatives of
$f(x) = x(\ln x)^2 - 2x \ln x + 2x$, and $g(x) = (x^2/2) \ln x - (x^2/4)$.

(ii) Let R_ϵ be the region bounded by the curves $x = \epsilon$ and $y = \ln x$, where $0 < \epsilon < 1$. Find the volume of the solid obtained by revolving R_ϵ around the x-axis.

(iii) Find the volume of the solid obtained by revolving R_ϵ around the y-axis.

Answer:

(i) $f'(x) = (\ln x)^2$ and $g'(x) = x \ln x$.

(ii)

$$V = \pi \int_\epsilon^1 (\ln x)^2 \, dx = \pi \left(f(x) \Big|_\epsilon^1 \right) = \pi (2 - \epsilon (\ln \epsilon)^2 + 2 \epsilon \ln \epsilon - 2\epsilon)$$

Observe that the limit as $\epsilon \to 0$ is 2π.

(iii)

$$V = 2\pi \left| \int_\epsilon^1 x \ln x \, dx \right| = 2\pi \left(g(x) \Big|_1^\epsilon \right) = \frac{\pi}{2} \left(1 + 2\epsilon^2 \ln \epsilon - \epsilon^2 \right).$$

Observe that the limit as $\epsilon \to 0$ is $\pi/2$.