



Quiz # 10  
Math 101-Section 01 Calculus I  
20 April, 2018, Friday  
Instructor: Ali Sinan Sertöz  
**Solution Key**



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**Q-1)**

(i) Find and simplify the derivatives of

$$f(x) = x(\ln x)^2 - 2x \ln x + 2x, \text{ and } g(x) = (x^2/2) \ln x - (x^2/4).$$

(ii) Let  $R_\epsilon$  be the region bounded by the curves  $x = \epsilon$  and  $y = \ln x$ , where  $0 < \epsilon < 1$ . Find the volume of the solid obtained by revolving  $R_\epsilon$  around the  $x$ -axis.

(iii) Find the volume of the solid obtained by revolving  $R_\epsilon$  around the  $y$ -axis.

**Answer:**

(i)  $f'(x) = (\ln x)^2$  and  $g'(x) = x \ln x$ .

(ii)

$$V = \pi \int_{\epsilon}^1 (\ln x)^2 dx = \pi \left( f(x) \Big|_{\epsilon}^1 \right) = \pi(2 - \epsilon (\ln \epsilon)^2 + 2\epsilon \ln \epsilon - 2\epsilon)$$

Observe that the limit as  $\epsilon \rightarrow 0$  is  $2\pi$ .

(iii)

$$V = 2\pi \left| \int_{\epsilon}^1 x \ln x dx \right| = 2\pi \left( g(x) \Big|_1^{\epsilon} \right) = \frac{\pi}{2} (1 + 2\epsilon^2 \ln \epsilon - \epsilon^2).$$

Observe that the limit as  $\epsilon \rightarrow 0$  is  $\pi/2$ .