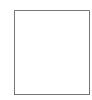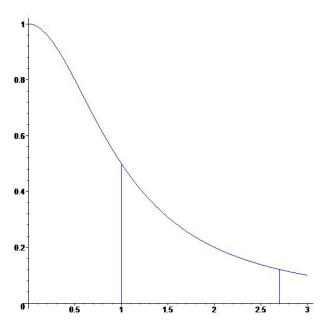Quiz # 10
Math 101-Section **13** Calculus I
13 December 2018, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1) For any real number $t > 0$, let $A(t)$ be the volume of the solid obtained by rotating the region under the curve $y = \dfrac{1}{x^2 + 1}$ and above the curve $y = 0$ from $x = t$ to $x = et$ around the y-axis. Find $\lim\limits_{t \to \infty} A(t)$.

Solution:

$$A(t) = 2\pi \int_t^{et} \frac{x}{x^2 + 1} \, dx = \pi \left(\ln(x^2 + 1) \Big|_t^{et} \right) = \pi \ln \frac{e^2 t^2 + 1}{t^2 + 1}.$$

Now it is clear that

$$\lim_{t \to \infty} A(t) = \pi \ln e^2 = 2\pi.$$