Bilkent University

Quiz # 01
Math 101-Section 08 Calculus I
3 October 2019, Thursday
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Solution Key

Q-1) A function f is defined as

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{ax+1}-1}{\sqrt{bx+4}-2} & x \neq 0, \\ \\ 1 & x = 0 \end{cases}$$

Where a and b are real numbers.

(i) Find all a and b such that f is continuous at $x = 0$. (6 points)

(ii) Find the domain of f if moreover $a = 1$. (4 points)

Solution:

(i) First we observe that f is not defined anywhere if $b = 0$, so we must take $b \neq 0$ to begin with. Putting $x = 0$ gives an indeterminate form, so we use algebra to simplify the expression of f when $x \neq 0$.

$$\begin{aligned} f(x) &= \frac{\sqrt[3]{ax+1}-1}{\sqrt{bx+4}-2} \cdot \frac{\sqrt[3]{ax+1}^2 + \sqrt[3]{ax+1}+1}{\sqrt[3]{ax+1}^2 + \sqrt[3]{ax+1}+1} \cdot \frac{\sqrt{bx+4}+2}{\sqrt{bx+4}+2} \\ &= \frac{ax}{bx} \cdot \frac{\sqrt{bx+4}+2}{\sqrt[3]{ax+1}^{2/3} + \sqrt[3]{ax+1}+1} \\ &= \frac{a}{b} \cdot \frac{\sqrt{bx+4}+2}{\sqrt[3]{ax+1}^{2/3} + \sqrt[3]{ax+1}+1}, \quad \text{(since } x \neq 0) \\ &\to \frac{a}{b} \cdot \frac{4}{3}, \quad \text{(as } x \to 0.) \end{aligned}$$

We now conclude that f is continuous at $x = 0$ if $\dfrac{4a}{3b} = 1$. Hence for any non-zero b, if we take $a = 3b/4$, the function f will be continuous at $x = 0$.

(ii) If $a = 1$, then from the previous calculation we find that $b = 4/3$. For the domain of f we must chose x such that $bx + 4 \geq 0$, or equivalently $x \geq -4/b = -3$. Hence the domain is $[-3, \infty)$.

Here is a graph of $y = f(x)$.