Bilkent University
Quiz \# 05
Math 101-Section 08 Calculus I
31 October 2019, Thursday
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## Solution Key

Q-1) The top of a ladder slides down a vertical wall at a rate of $5 \mathrm{~m} / \mathrm{min}$. At the moment when the bottom of the ladder is 12 m from the wall, it slides away from the wall at a rate of $7 \mathrm{~m} / \mathrm{min}$. How long is the ladder?

## Solution:



In the above figure, $x$ and $y$ are functions of time. We are given the following facts:

$$
\begin{equation*}
y^{\prime}(t)=-5 \mathrm{~m} / \min , \quad x^{\prime}\left(t_{0}\right)=7 \mathrm{~m} / \min , \quad x\left(t_{0}\right)=12 \mathrm{~m} . \tag{*}
\end{equation*}
$$

From the figure we see that

$$
\begin{equation*}
x^{2}(t)+y^{2}(t)=L^{2} \tag{**}
\end{equation*}
$$

Taking derivatives of both sides of this equation and setting $t=t_{0}$, we get (after cancelling out 2 )

$$
x^{\prime}\left(t_{0}\right) x\left(t_{0}\right)+y^{\prime}\left(t_{0}\right) y\left(t_{0}\right)=0
$$

Putting in the values from (*) we get

$$
(7 m / \min )(12 m)+(-5 m / \min ) y\left(t_{0}\right)=0
$$

which gives

$$
y\left(t_{0}\right)=\frac{84}{5} m
$$

Putting in the values of $x\left(t_{0}\right)$ and $y\left(t_{0}\right)$ into $(* *)$, we find

$$
(12 m)^{2}+\left(\frac{84}{5} m\right)^{2}=L^{2}
$$

which gives

$$
L=\frac{12 \sqrt{74}}{5} m \approx 20 m 65 \mathrm{~cm}
$$

