Bilkent University

Quiz # 09
Math 101-Section 08 Calculus I
28 November 2019, Thursday
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Solution Key

Q-1) Find a rational function of the form $f(x) = \dfrac{ax^2+b}{cx^3+d}$, where a, b, c, d are integers, such that

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{i^2+n^2}{i^3+2n^3} = \int_0^5 f(x)\, dx.$$

(Do not attempt to evaluate the integral! Its value, and hence the limit, is $0.5859\ldots$*.)*

Solution:

Subdivide the interval $[0,5]$ into n equal subintervals. The length of each subinterval is $5/n$, and the location of the i-th subdivision is $5i/n$, $i = 0,\ldots,n$. We now rewrite the expression $\dfrac{i^2+n^2}{i^3+2n^3}$ as

$$\frac{i^2+n^2}{i^3+2n^3} = \frac{\left(\frac{i}{n}\right)^2+1}{\left(\frac{i}{n}\right)^3+2} \cdot \frac{1}{n} = \frac{\frac{1}{25}\left(\frac{5i}{n}\right)^2+1}{\frac{1}{125}\left(\frac{5i}{n}\right)^3+2}\ \frac{1}{5}\ \frac{5}{n}.$$

Seeing $5/n$ as Δx and $5i/n$ as x, we conclude that

$$\frac{1}{5}\cdot\frac{i^2+n^2}{i^3+2n^3} = \frac{\frac{1}{25}(x)^2+1}{\frac{1}{125}(x)^3+2}\ \Delta x = \frac{x^2+25}{x^3+250}\ \Delta x.$$

Hence

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{i^2+n^2}{i^3+2n^3} = \int_0^5 \frac{x^2+25}{x^3+250}\, dx.$$