Q-1) Consider the function $f(x) = \dfrac{x^2 + 1}{x}$.

    (i) Find $f'(x)$ and its roots, and sign changes.

    (ii) Find $f''(x)$ and its roots, and sign changes.

    (iii) Find vertical, horizontal and slant asymptotes, if exists.

    (iv) Sketch the graph of $y = f(x)$, indicating clearly the concavity of the graph.

Solution: **(i)** $f'(x) = \dfrac{x^2 - 1}{x^2} = 0$ when $x = \pm 1$.

$f'(x) > 0$ on $(-\infty, -1)$ and on $(1, \infty)$. It is negative elsewhere except at $x = 0$ where it is not defined.

(ii) $f''(x) = \dfrac{2}{x^3}$.

$f''(x)$ is never zero. It is positive when $x > 0$ and negative when $x < 0$.

(iii) $f(x) = x + \dfrac{1}{x}$, hence $y = x$ is a slant asymptote, and $x = 0$ is a vertical asymptote. There is no horizontal asymptote.

(iv) Here is the graph of $y = f(x)$, together with $y = x$.