Bilkent University

Quiz # 09
Math 101-Section 12 Calculus I
13 December 2020 Sunday
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Solution Key

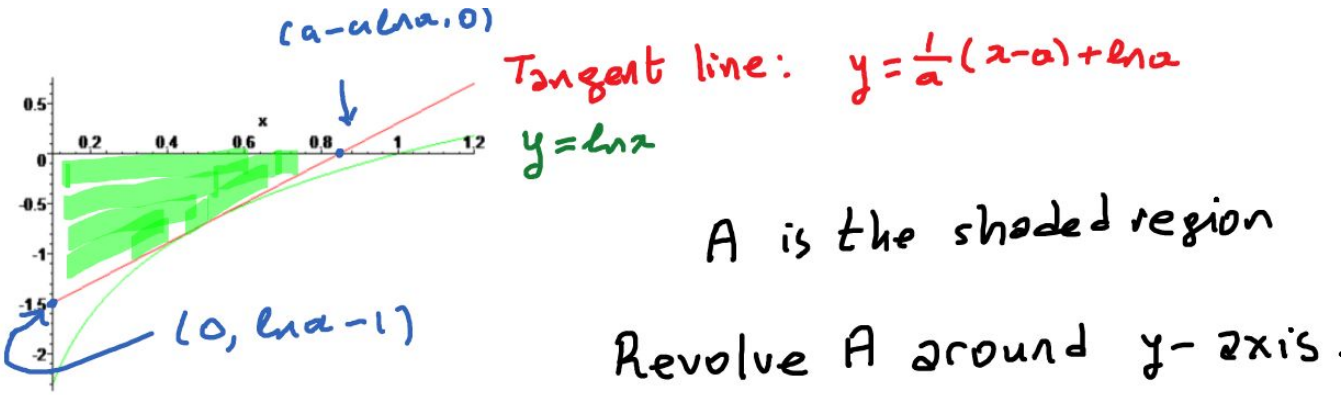
Q-1) Let L be the line tangent to $y = \ln x$ at the point a where $0 < a < 1$, and let A denote the region bounded by the line L and the axes.

(a) Find the volume of the solid obtained by revolving the region A around the y-axis using the disk method.

(b) Find the volume of the solid obtaned by revolving the region A around the y-axis using the shell method.

(c) Now put $a = 1/2$ in both of the results you obtained above and calcıulate the result using a calculator. Did you get the same result? Did you get a negative volume? Correct your results if you got some discrepancies!

Solution:



The equation of the tangent line is $y = (1/a)(x - a) + \ln a$. It intersects x-axis at $(a - a\ln a, 0)$, and y-axis at $(0, \ln a - 1)$.

(a) The distance from y-axis to L is $x = ay - a\ln a + a$ at the point $y \in [\ln a - 1, 0]$. Note here that we are writing the interval in the increasing direction. If you write $[0, \ln a - 1]$ you will get a negative volume!

$$V = \pi \int_{\ln a - 1}^{0} (ay - a\ln a + a)^2 \, dy = \pi \left(\frac{\pi \, (ay - a\ln a + a)^3}{3a} \Big|_{\ln a - 1}^{0} \right)$$

$$= \pi a^2 \left(-\frac{1}{3}(\ln a)^3 + (\ln a)^2 - \ln a + \frac{1}{3} \right).$$

(b) Observe that $y = \dfrac{1}{a}[x - (a - a\ln a)]$ and $0 \le x \le a - a\ln a$. Therefore $y < 0$ on this interval. For the shell method we need the height of the shell, which is $|y| = -y$.

$$\begin{aligned} V &= 2\pi \int_0^{a-a\ln a} x \left\{ -\frac{1}{a}[x - (a - a\ln a)] \right\} \, dx \\ &= \pi \left(\frac{x^2\,(2\,x + 3\,a\ln a - 3\,a)}{a} \Big|_0^{a-a\ln a} \right) \\ &= \frac{\pi}{3}(1 - \ln a)^3 a^2. \end{aligned}$$

(c) When we put $a = 1/2$ in both of the expressions above we get

$$V = 1.270728476...$$