Bilkent University

Quiz # 01
Math 101-Section 12 Calculus I
07 October 2021 Thursday
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**Solution Key**

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**Q-1)** Let $f$ be defined as

$$f(x) = \begin{cases} x^3 + x, & x \geq 0 \\ x^2 - x, & x < 0 \end{cases}$$

(i) Is $f$ continuous at $x = 0$

(ii) Is $f$ differentiable at $x = 0$?

(iii) Write an equation for the tangent line to the curve $y = f(x)$ at $x = 1$.

(iv) Write an equation for the tangent line to the curve $y = f(x)$ at $x = -\dfrac{1}{2}$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: (i) 2 points, (ii) 2 points, (iii) 3 points, (iv) 3 points.

**Solutions:**

**(i)**

$$\begin{aligned} \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} (x^2 - x) = 0 \\ \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} (x^3 + x) = 0 \\ f(0) &= 0. \end{aligned}$$

The left and right limits of $f$ at $x = 0$ agree and both are equal to $f(0)$, therefore $f$ is continuous at $x = 0$.

**(ii)**

$$\begin{aligned} \lim_{x \to 0^-} \frac{f(x) - f(0)}{x} &= \lim_{x \to 0^-} \frac{x^2 - x}{x} = \lim_{x \to 0^-} (x - 1) = -1 \\ \lim_{x \to 0^+} \frac{f(x) - f(0)}{x} &= \lim_{x \to 0^+} \frac{x^3 + x}{x} = \lim_{x \to 0^+} (x^2 + 1) = 1. \end{aligned}$$

The left and right limits of the Newton quotient do not agree, so

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x} \quad \text{does not exist.}$$

Hence $f$ is not differentiable at $x = 0$.

**(iii)** $f(1) = 2$, $f'(1) = 3x^2 + 1\Big|_{x=1} = 4$. Hence an equation of the tangent line is

$$y - 2 = 4(x - 1).$$

**(iv)** $f(-1/2) = -3/4$, $f'(-1/2) = 2x - 1\Big|_{x=-\frac{1}{2}} = -2$. Hence an equation of the tangent line is

$$y + \frac{3}{4} = -2(x + \frac{1}{2}1).$$