Bilkent University

Quiz # 05
Math 101-Section 12 Calculus I
11 November 2021 Thursday
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**Solution Key**

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**Q-1)** Let $f(x) = x^4 - 2x^3 - 12x^2$ where $-4 \leq x \leq 6$.

(a) Find the $x$-intercepts of $f(x)$.

(b) Find the critical points of $f(x)$.

(c) Find the inflection points of $f(x)$.

(d) Plot $y = f(x)$ where $-4 \leq x \leq 6$.

(e) By looking at your graph find the points where $f$ takes its minimum and maximum values.

Show your work. Simplify as much as possible.

Grading: 2 points each

**Solutions:**

**(a)** $f(x) = x^4 - 2x^3 - 12x^2 = x^2(x^2 - 2x - 12) = 0$ gives

$$x = 0, x = 1 + \sqrt{13} \approx 4.6, x = 1 - \sqrt{13} \approx -2.6.$$

**(b)** $f'(x) = 4x^3 - 6x^2 - 24x = 2x(2x^2 - 3x - 12) = 0$ gives

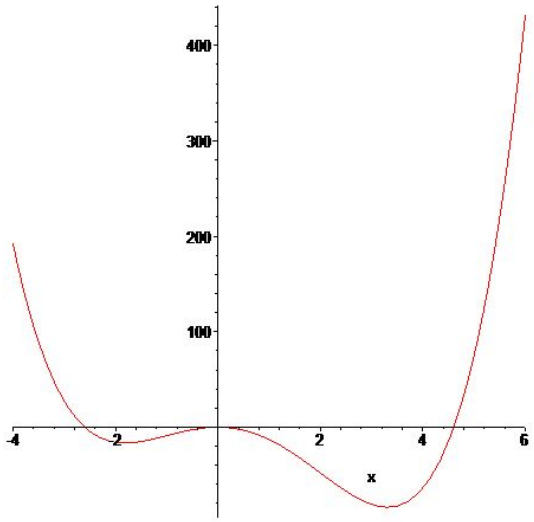
$$x = 0, x = \frac{3 + \sqrt{105}}{4} \approx 3.3, x = \frac{3 - \sqrt{105}}{4} \approx -1.8.$$

**(c)** $f''(x) = 12x^2 - 12x - 24 = 12(x^2 - x - 2) = 12(x + 1)(x - 2)$ gives

$$x = -1, x = 2.$$

Since $f''(x)$ changes sign at each of these roots, both are inflection points.

**(d)** Here is a graph of $y = f(x)$ when $-4 \leq x \leq 6$.




**(e)** By looking at the graph we see that the minimum of $f$ occurs at the critical point $x = 1 + \sqrt{13}$, and the maximum occurs at the end point $x = 6$.