



Bilkent University

Quiz # 07  
Math 101-Section 12 Calculus I  
24 November 2022 Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

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**Q-1)** Find  $y$  which is a function of  $x$  with  $y(5) = 3$  and satisfies the differential equation

$$y' = -\frac{1 + y^2}{1 + x^2}, \quad x \neq \frac{7}{4}.$$

*Show your work in detail. Correct answers without detailed explanation do not get any credit.*

Grading: 10 points.

Hint:  $\int \frac{dx}{1+x^2} = \arctan x + C$ , and  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ .

**Solution:**

$$\frac{dy}{dx} = -\frac{1 + y^2}{1 + x^2}$$

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$$\arctan y = -\arctan x + C.$$

This gives  $\arctan y + \arctan x = C$ . Taking  $\tan$  of both sides gives

$$\frac{y + x}{1 - yx} = C,$$

where  $C$  is still an arbitrary constant. Now putting in  $x = 5$  and  $y = 3$  we get

$$C = -\frac{4}{7},$$

and solving for  $y$  with this value of  $C$  gives

$$y = -\frac{7x + 4}{7 - 4x}, \quad \text{where we must have } x \neq \frac{7}{4}.$$