Quiz \# 03
Math 101-Section 05 Calculus I
12 October 2023 Thursday
Instructor: Ali Sinan Sertöz

## Solution Key

## Q-1)

(a) Let $f(x)=\sin \left(\cos ^{2}\left[\left(x^{2}+1\right)^{7}\right]\right)$. Calculate $f^{\prime}(x)$. Do not simplify!
(b) Let $y$ be a differentiable function of $x$ satisfying $x^{4}+y^{7}+x y+1=20 y^{3}$. Write an equation for the tangent line of the curve defined by this equation at the point $(x, y)=(2,1)$

Show your work in detail unless asked otherwise. Correct answers without detailed explanation do not get any credit.
Grading: 5+5=10

## Solution:

(a)

$$
f^{\prime}(x)=\cos \left(\cos ^{2}\left[\left(x^{2}+1\right)^{7}\right]\right) \cdot 2 \cos \left[\left(x^{2}+1\right)^{7}\right] \cdot\left(-\sin \left[\left(x^{2}+1\right)^{7}\right]\right) \cdot\left(7\left(x^{2}+1\right)^{6}\right) \cdot(2 x)
$$

(b) Implicitly differentiating the given equation we get

$$
4 x^{3}+7 y^{6} y^{\prime}+y+x y^{\prime}=60 y^{2} y^{\prime}
$$

Putting in $x=2$ and $y=1$ we find that $y^{\prime}=11 / 17$.
Hence an equation for the tangent line at $(2,1)$ is

$$
y=\frac{11}{17}(x-2)+1
$$

