Math 102 Calculus II
Quiz-2
Solutions

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Sec01-1) Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^3+x^2y+y^3}{x^2+xy+y^2}$, if the limit exists.

Solution: Substitute $x = r\cos t$, $y = r\sin t$ to obtain

$$\frac{r\,(\cos^3 t + \sin t)}{\sin t\cos t + 1}.$$

This goes to zero as r goes to zero, regardless of t. So the limit exists and is 0.

Sec01-2) Let $f(x,y) = x^3 + x^7y + xy^8 + y^3$, $x(t) = 5\sin t + \cos t$, $y(t) = 2\tan t + \sec t$. If we define $g(t) = f(x(t), y(t))$, calculate $g'(0)$.

Solution: $x(0) = 1$, $y(0) = 1$, $x'(t) = 5\cos t - \sin t$, $y'(t) = 2\sec^2 t + \sec t\tan t$, $x'(0) = 5$, $y'(0) = 2$.
$f_x = 3x^2 + 7x^6y + y^8$, $f_y = x^7 + 8xy^7 + 3y^2$, $f_x|_{t=0} = 11$, $f_y|_{t=0} = 12$.
$g'(0) = 11\cdot 5 + 12\cdot 2 = 79$.

Sec02-1) Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^4+xy+y^2}{y^4+x^2y^2+x^2}$, if the limit exists.

Solution: Substitute $x = r\cos t$, $y = r\sin t$ to obtain

$$\lim_{(x,y)\to(0,0)} \frac{r^2\cos^4 t + \cos t \sin t + \sin^2 t}{r^2\sin^4 t + r^2\cos^2 t \sin^2 t + \cos^2 t} = \frac{\cos t \sin t + \sin^2 t}{\cos^2 t},$$

which depends on t, so the limit does not exist.

Sec02-2) Let $f(x,y) = x^6+x^7y^5+x^5y^7+14xy+y^17$, where $x(t) = 2t^2+4t+1$, $y(t) = t^2+2t$. If we define $g(t) = f(x(t), y(t))$, then find $g'(0)$.

Solution: $x(0) = 1$, $y(0) = 0$, $x'(t) = 4t+4$, $y'(t) = 2t+2$, $x'(0) = 4$, $y'(0) = 2$.
$f_x = 6x^5+7x^6y^5+5x^4y^7+14y$, $f_y = 5x^7y^4+7x^5y^6+14x+17y^16$, $f_x|_{t=0} = 6$, $f_y|_{t=0} = 14$.
$g'(0) = 6\cdot 4 + 14\cdot 2 = 52$.
