Math 102 Calculus – Homework 1 – Solutions
Due on 14 July 2006 Friday, class time

The first question is 10 points, the others are 15 points.

Q-1 Evaluate the integral $\displaystyle\int_0^2 \int_x^2 2y^2 \sin(xy)\, dy\, dx.$

Solution: $\displaystyle\int_0^2 \int_x^2 2y^2 \sin(xy)\, dy\, dx = \int_0^2 \int_0^y 2y^2 \sin(xy)\, dx\, dy = \int_0^2 \Big(-2t\cos(xy)\Big|_0^y \Big)\, dy$

$\displaystyle= \int_0^y (-2y\cos(xy) + 2y)\, dy = \Big(-\sin y^2 + y^2 \Big|_0^2 \Big) = 4 - \sin 4.$

Q-2 Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.

Solution:

$$\text{Volume} = \int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{3-y} dz\, dx\, dy = \int_0^2 \int_0^{\sqrt{4-y^2}} (3-y)\, dx\, dy = \int_0^{\pi/2} \int_0^2 (3 - r\sin\theta) r dr\, d\theta$$
$$= \int_0^{\pi/2} \left(6 - \frac{8}{3}\sin\theta\right) d\theta = 3\pi - \frac{8}{3}.$$

Q-3 Evaluate the integral $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx.$

Solution:

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx = \int_0^{2\pi} \int_0^1 \frac{2r\, dr\, d\theta}{(1+r^2)^2} = \int_0^{2\pi} \int_1^2 \frac{du\, d\theta}{u^2} = \pi.$$

Q-4 Find the volume of the wedge cut from the cylinder $x^2+y^2=1$ by the planes $z=y$ and $z=3y$.

Solution:

$$\text{Volume} = \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{y}^{3y} dz\ dy\ dx = \int_{0}^{\pi}\int_{0}^{1}(2r\sin\theta)r\ dr\ d\theta = \frac{4}{3}.$$

Q-5 Evaluate the integral $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\ dy\ dx\ dz.$

Solution:

$$\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\ dy\ dx\ dz = \int_0^1\int_0^1\int_0^{\sqrt{y}} 12xze^{zy^2}\ dx\ dy\ dz = \int_0^1\int_0^1 6yze^{zy^2}\ dy\ dz = \int_0^1 3(e^z-1)\ dz = 3(e-2).$$

Q-6 Find the volume of the cap cut from the sphere $x^2+y^2+z^2=R^2$ by the plane $z=h$, where $0\le h\le R$.

Solution:

$$V(h) = \int_0^{2\pi}\int_0^{\sqrt{R^2-h^2}}\int_h^{\sqrt{R^2-r^2}} dz\ r\ dr\ d\theta$$

$$= 2\pi\int_0^{\sqrt{R^2-h^2}}\left(r\sqrt{R^2-r^2}-rh\right)dr = 2\pi\left(\int_h^R u^2\ du - \frac{h}{2}(R^2-h^2)\right) = \left(\frac{2}{3}R^3 - R^2h + \frac{1}{3}h^3\right)\pi.$$

Q-7 Use the transformation $u=x-y$, $v=2x+y$ to evaluate the integral $\displaystyle\int\int(2x^2-xy-y^2)\ dx\ dy$ for the region R in the first quadrant bounded by the lines $y=-2x+4$, $y=-2x+7$, $y=x-2$, and $y=x+1$.

Solution:

$$\int\int_R(2x^2-xy-y^2)\ dx\ dy = \frac{1}{3}\int_4^7\int_{-1}^2 uv\ du\ dv = \frac{1}{3}\left(\frac{u^2}{2}\Big|_{-1}^{2}\right)\left(\frac{v^2}{2}\Big|_{4}^{7}\right) = \frac{33}{4}.$$

Please send comments and questions to `sertoz@bilkent.edu.tr`