Math 102 Calculus II – Homework II

Due on July 13, 2007 Friday 17:00

Q-1) Let $\mathbf{F} = x^2\,\mathbf{i} + z\,\mathbf{j} + yz\,\mathbf{k}$, and C the curve parametrized as $\mathbf{r} = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t^2\,\mathbf{k}$ for $0 \le t \le \pi$. Evaluate the work integral

$$\int \mathbf{F} \cdot \mathbf{T}\,ds$$

on the curve C.

Q-2) Find a potential function $f(x, y, z)$ for the field $\mathbf{F} = \ln x\,\mathbf{i} + \cos(y+z)\,\mathbf{j} + (z + \cos(y+z))\,\mathbf{k}$ such that $f(1, \frac{\pi}{2} - 1, 1) = -1$.

Q-3) Find the area enclosed by the simple curve C parametrized as $\mathbf{r}(t) = t^4\,\mathbf{i} + (t - t^3)\,\mathbf{j}$ for $-1 \le t \le 1$. May I remind you that area is and should be a non-negative number. If you find a negative number, you owe an explanation!

Q-4) Let R_α be the region in the xz-plane bounded by the lines $z = \alpha x$, $z = 1$ and $x = 0$, where $\alpha \ge 1$. Let $A(\alpha)$ denote the area of the surface $z^2 = x^2 + y^2$ lying above R_α. First, without doing any calculations, find $A(1)$ and $\lim_{\alpha \to \infty} A(\alpha)$. Then calculate $A(\alpha)$ explicitly in terms of α. Check your answer with what you found above.

Q-5) Let $\mathbf{F} = x \ln(1 + z^2)\,\mathbf{i} + y \tan z \cos x\,\mathbf{j} + z \ln(4 + x^4 + y^4)\,\mathbf{k}$ be a field defined on the hemisphere S given by $x^2 + y^2 + z^2 = 1$, $z \ge 0$. Calculate explicitly

$$\int\!\!\int_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n}\,d\sigma.$$

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