Date: July 4, 2008, Friday NAME:..

Time: 9:30-11:30

Ali Sinan Sertöz STUDENT NO:..

Math 102 Calculus II – Midterm Exam II – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

PLEASE READ:

Check that there are 5 questions on your exam booklet. Write your name on the top of every page. Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Q-1) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^4+x^2y^2+y^4}$.

Solution:

First observe that

$$\left|\frac{x^3y^2}{x^4+x^2y^2+y^4}\right| = \frac{(x^2y^2)|x|}{x^4+x^2y^2+y^4} \leq \frac{(x^4+x^2y^2+y^4)|x|}{x^4+x^2y^2+y^4} \leq |x|.$$

Then conclude by the sandwich theorem that the required limit is zero.

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Q-2) Let $\omega = (x + 2y + 3z)^{15}$ where $x = (u - v)^4 - 16$, $y = \cos^5(u + v) - 2$ and $z = \ln(u^2 + v^2) - \ln 2 + 1$. Find $\dfrac{\partial \omega}{\partial u}$ at the point $(u, v) = (-1, 1)$.

Solution:

Using chain rule, first write $\omega_u = \omega_x x_u + \omega_y y_u + \omega_z z_u$. Then observe that $(x, y, z) = (0, -1, 1)$ at the point $(u, v) = (-1, 1)$. Putting these values in, you will find $\omega_u = -525$.

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Q-3) Find the directional derivative of z in the direction of $(5, 12)$ at the point $(x, y) = (-1, 1)$ if z is defined as a differentiable function of x and y at the point $(x, y, z) = (-1, 1, 0)$ by the equation $x^2y + e^{yz} + 2xz = 2$.

Solution:

By implicit differentiation you first find $z_x = -2$ and $z_y = 1$ at the point $(-1, 1, 0)$. Then the required directional derivative is $(-2, 1) \cdot (5/13, 12/13) = 2/13$.

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Q-4) Find and classify all the critical points of $f(x, y) = x^3 + y^2 + x^2y$.

Solution:

$f_x = 0$ and $f_y = 0$ give $(0, 0)$ and $(3, -9/2)$ as the critical points.

At $(0, 0)$ the discriminant is zero. But $f(x, 0) = x^3$ takes both positive and negative values in every neighborhood of the origin, so the origin is a saddle point.

At the other critical point the discriminant is negative so it is also a saddle point.

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Q-5) Find the points on the surface $z^3 + x^2y = 1$ closest to the origin.

It may be necessary to notice that $(108/31)^{2/3} > 22/10$, *and that* $(16)^{1/3} > 2$.

Solution:

Using the Lagrange multipliers method you will find that the points $(0, 0, 1)$, $(\pm 4^{1/3}/\sqrt{2}, 4^{1/3}/2, 0)$ and $(\pm c/\sqrt{2}, c/2, c/3)$, where $c^3 = 108/31$ are the critical points. A brief comparison shows that the second and third sets of points have a distance larger than 1 from the origin. So the point $(0, 0, 1)$ is the point on the surface closest to the origin.