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Math 102 Calculus II - Homework I

| 1 | 2 | 3 | 4 | TOTAL |
| :---: | :---: | :---: | :---: | :---: |
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|  |  |  |  |  |
| 25 | 25 | 25 | 25 | 100 |

Please do not write anything inside the above boxes!

## PLEASE READ:

Check that there are 4 questions on your booklet. Write your name on the top of every page. Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Q-1) Consider the function

$$
f(x, y)= \begin{cases}\frac{x^{5}+y^{6}}{\left(x^{2}+y^{2}\right)^{\alpha}} & \text { if }(x, y) \neq(0,0) \\ 0 & \text { if }(x, y)=(0,0)\end{cases}
$$

Find all value of $\alpha \in \mathbb{R}$ such that both $f_{x}(0,0)$ and $f_{y}(0,0)$ exist. Calculate $f_{x}(0,0)$ and $f_{y}(0,0)$ for all such values of $\alpha$.

Q-2) Assume that $3 z+x+y^{2}+x z^{3}=13$ defines $z$ as a $C^{2}$ function of $x$ and $y$ around the point $(x, y, z)=(3,2,1)$. Find the values of $z_{x}, z_{y}, z_{x y}, z_{y x}, z_{x x}$ and $z_{y y}$ at the point $(x, y, z)=(3,2,1)$.

Q-3) Let $S$ be the surface in $\mathbb{R}^{3}$ given by $f(x, y, z)=0$ where $f(x, y, z)=1+x^{2}+y^{4}-z$. Let $p_{0}=\left(1 / 2, y_{0}, z_{0}\right)$ be a point on the surface such that the tangent plane to the surface $S$ at $p_{0}$ passes through the origin. Find $z_{0}$.

Q-4) Let $F(x)=\int_{x^{4}}^{x^{3}} \sqrt{t^{3}+x^{2}} d t$. Calculate $F^{\prime}(x)$ and find explicitly the values of $F^{\prime}(0)$ and $F^{\prime}(1)$.

Hint: Assume that you can differentiate under the integral sign; see the last few problems at the end of the section on "The Chain Rule" of Thomas' Calculus.

