



Quiz # 2
 Math 102-Section 06 Calculus II
 23 February 2017, Thursday
 Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Student ID:

Your Department:

Show your work in detail. Correct answers without justification are never graded.

Q-1) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{xy \tan(x^2 + y^2)}{x^4 + y^2}$, if the limit exists. Show your work in a readable form! (10 points)

Answer: First we notice that

$$\begin{aligned} \frac{xy \tan(x^2 + y^2)}{x^4 + y^2} &= \frac{xy(x^2 + y^2)}{x^4 + y^2} \frac{\sin(x^2 + y^2)}{(x^2 + y^2)} \frac{1}{\cos(x^2 + y^2)} \\ &= \left(\frac{x^3y}{x^4 + y^2} + \frac{xy^3}{x^4 + y^2} \right) \frac{\sin(x^2 + y^2)}{(x^2 + y^2)} \frac{1}{\cos(x^2 + y^2)}. \end{aligned}$$

Now we check separately that

$$\begin{aligned} \lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^4 + y^2} &= 0 \text{ by Sertöz Theorem } \odot \\ \lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^4 + y^2} &= 0 \text{ by Sertöz Theorem } \odot \\ \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{(x^2 + y^2)} &= 1 \text{ Elementary!} \\ \lim_{(x,y) \rightarrow (0,0)} \frac{1}{\cos(x^2 + y^2)} &= 1 \text{ Elementary!} \end{aligned}$$

Putting these together we find that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy \tan(x^2 + y^2)}{x^4 + y^2} = 0.$$