Quiz # 7
Math 102-Section **06** Calculus II
6 April 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Let $V(R,\alpha)$ be the volume of the solid bounded from above by $x^2+y^2+z^2=R^2$ and from below by $z=(\tan\alpha)\sqrt{x^2+y^2}$. Here $R>0$ and $0\le\alpha<\pi/2$.

Show that $V(R,\alpha)=\dfrac{2\pi R^3}{3}(1-\sin\alpha)$.

Answer:

It is best to set up the volume integral in spherical coordinates.

$$\begin{aligned}
V(R,\alpha) &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}-\alpha}\int_0^{R}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta \\
&= \left(\int_0^{2\pi}d\theta\right)\left(\int_0^{\frac{\pi}{2}-\alpha}\sin\phi\,d\phi\right)\left(\int_0^{R}\rho^2\,d\rho\right) \\
&= \left(\theta\Big|_0^{2\pi}\right)\left(-\cos\phi\Big|_0^{\frac{\pi}{2}-\alpha}\right)\left(\frac{1}{3}\rho^3\Big|_0^{R}\right) \\
&= (2\pi)\left(1-\cos(\frac{\pi}{2}-\alpha)\right)\left(\frac{1}{3}R^3\right) \\
&= \frac{2\pi R^3}{3}(1-\sin\alpha).
\end{aligned}$$