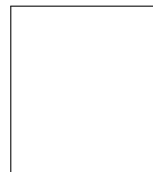




Quiz # 9
Math 102-Section 06 Calculus II
20 April 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Student ID:

Your Department:

Show your work in detail. Correct answers without justification are never graded.

Q-1) Let $a_n = \frac{n+4}{n(n+1)(n+2)}$.

(i) Show that $\sum_{n=1}^{\infty} a_n$ converges.

(ii) Find the exact value of $\sum_{n=1}^{\infty} a_n$.

Answer:

(i) Let $b_n = \frac{1}{n^2}$. Using Limit Comparison Test we find that

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1,$$

so $\sum_{n=1}^{\infty} a_n$ converges since $\sum_{n=1}^{\infty} b_n$, which is a p -series, converges.

(ii) Using partial fractions technique we find that

$$\frac{n+4}{n(n+1)(n+2)} = \frac{2}{n} - \frac{3}{n+1} + \frac{1}{n+2}.$$

Using this we find that the sequence of partial sums s_m is of the form

$$s_m = a_1 + \cdots + a_m = \frac{3}{2} - \frac{m+3}{(m+1)(m+2)}.$$

Hence

$$\sum_{n=1}^{\infty} a_n = \lim_{m \rightarrow \infty} s_m = \frac{3}{2}.$$