Bilkent University

Quiz # 01
Math 102-Section **10** Calculus II
14 February 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Find the unit tangent vector of the parametric curve $\vec{r}(t) = (t^2 - t, t - 2, t - 2t^2)$, $t \in \mathbb{R}$, at the point where it intersects the plane that passes through the points $\vec{a} = (7, 8, 8)$, $\vec{b} = (11, 13, 15)$ and $\vec{c} = (8, 10, 12)$.

Grading: Equation of the plane: 4, intersection point: 3, tangent vector there: 2, unit tangent vector 1. Total: 10 points.

Solution:

We first find two vectors along the plane. The easiest ones to find are

$$\vec{u}_1 = \vec{b} - \vec{a} = (4, 5, 7), \text{ and } \vec{u}_2 = \vec{c} - \vec{a} = (1, 2, 4).$$

Next we find a vector orthogonal to these.

$$\vec{w} = \vec{u}_1 \times \vec{u}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 7 \\ 1 & 2 & 4 \end{vmatrix} = (6, -9, 3).$$

Note that $\vec{w} \cdot \vec{p}$ is constant for all points $\vec{p} = (x, y, z)$ on this plane. This tells us that an equation for this plane is

$$\vec{w} \cdot \vec{p} = \vec{w} \cdot \vec{a}, \text{ or equivalently } 6x - 9y + 3z = -6. \qquad \textbf{(4 pts)}$$

To find the point of intersection of the given curve with this plane, we substitute its parametrization into the above equation of the plane to get

$$6(t^2 - t) - 9(t - 2) + 3(t - 2t^2) = -6, \text{ which gives } t = 2. \qquad \textbf{(3 pts)}$$

We then have

$$\vec{v}(t) = \vec{r}(t)' = (2t - 1, 1, 1 - 4t), \vec{v}(2) = (3, 1, -7) \text{ and } |\vec{v}(2)| = \sqrt{59}. \qquad \textbf{(2 pts)}$$

Finally, the unit tangent vector at that point is

$$\vec{T}(2) = (\frac{3}{\sqrt{59}}, \frac{1}{\sqrt{59}}, -\frac{7}{\sqrt{59}}). \qquad \textbf{(1 pt)}$$