Bilkent University

Quiz # 03
Math 102-Section **10** Calculus II
28 February 2019, Thursday
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Solution Key

Q-1) Let $f(x,y,z) = x^3 + x^2y^2 + yz^4 + z^3$ and let S be the surface in $\mathbb{R}^3$ defined by the equation $f(x,y,z) = 6$. Let $p_0 = (1,2,-1)$ be a point on the surface.

(i) Assuming that z is given as a differentiable function of x, y on S around p_0, write a linearization of z at p_0 in the form $z = L_1(x,y)$.

(ii) Assuming that y is given as a differentiable function of x, z on S around p_0, write a linearization of y at p_0 in the form $y = L_2(x,z)$.

(iii) Using the gradient of f at p_0, write an equation for the tangent plane of S at p_0.

(iv) Explain how the equation of the tangent line is related to the equations $z = L_1(x,y)$ and $y = L_2(x,z)$.

Grading: (i) 3 points, (ii) 3 points, (iii) 2 points, (iv) 2 points.

Solution:

(i)

$$z_x(p_0) = -\left.\frac{x(3x+2y^2)}{z^2(4yz+3)}\right|_{p_0} = \frac{11}{5},\ z_y(p_0) = -\left.\frac{2x^2y+z^4}{z^2(4yz+3)}\right|_{p_0} = 1,$$

so the linearization is

$$z = L_1(x,y) = \frac{11}{5}(x-1) + (y-2) - 1 = \frac{11}{5}x + y - \frac{26}{5}.$$

(ii)

$$y_x(p_0) = -\left.\frac{x(3x+2y^2)}{2x^2y+z^4}\right|_{p_0} = -\frac{11}{5},\ y_z(p_0) = -\left.\frac{z^2(4yz+3)}{2x^2y+z^4}\right|_{p_0} = 1,$$

so the linearization is

$$y = L_2(x,z) = -\frac{11}{5}(x-1) + (z+1) + 2 = -\frac{11}{5}x + z + \frac{26}{5}.$$

(iii)

$$\nabla f(p_0) = (3x^2 + 2xy^2, 2x^2y + z^4, 4yz^3 + 3z^2)\Big|_{p_0} = (11, 5, -5).$$

Hence an equation of the tangent plane is

$$11(x-1) + 5(y-2) - 5(z+1) = 0,$$

or equivalently

$$11x + 5y - 5z = 26.$$

(iv) If you solve the last equation for z, you get $L_1(x,y)$, and if you solve it for y, you get $L_2(x,z)$.