Bilkent University

Quiz # 01
Math 102 - Calculus II - Section 03
10 February 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) In the following find the infinite sums explicitly if they exist and expalin why if they don't exist.

(a) $\displaystyle\sum_{n=2}^{\infty} 2^n 3^{1-n}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{n(n+1)(n+2)}$.

Grading: 5+5 points

Solutions:

(a)

$$\sum_{n=2}^{\infty} 2^n 3^{1-n} = 3\sum_{n=2}^{\infty}\left(\frac{2}{3}\right)^n = 3\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^{n+2} = \frac{4}{3}\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n = \frac{4}{3}\frac{1}{1-(2/3)} = 4.$$

(b) First we apply partial fractions method to the general term to obtain

$$\frac{n+3}{n(n+1)(n+2)} = \frac{(3/2)}{n} - \frac{2}{n+1} + \frac{(1/2)}{n+2}.$$

The sequence of partial sums then becomes

$$s_n = \sum_{k=1}^{n} \frac{k+3}{k(k+1)(k+2)} = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}.$$

Hence

$$\sum_{n=1}^{\infty} \frac{n+3}{n(n+1)(n+2)} = \lim_{n\to\infty} s_n = \frac{5}{4}.$$