Bilkent University

Quiz # 7
Math 102-Section 09
5 May 2023, Friday, Moodle Quiz
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Solution Key

Q-1) Let $z = f(x, y)$ where f is a differentiable function. We restrict this function to the curve with parametric equations $(1 + 3t + t^2, t^4 - 1)$, $t \in \mathbb{R}$.
We also know that:

$$\begin{array}{lll} f_x(15,11) = 1, & f_x(11,15) = 3, & f_y(15,11) = 7 \\ f_y(11,15) = 11, & f_y(19,80) = 13, & f_y(80,11) = 17 \\ f_x(11,80) = 10, & f_x(80,11) = -1, & f_x(19,80) = -7 \\ f_y(11,80) = -6, & f_x(15,80) = 11, & f_y(15,80) = 16 \end{array}$$

Calculate $\left.\dfrac{\partial f}{\partial t}\right|_{t=2}$ and $\left.\dfrac{\partial f}{\partial t}\right|_{t=3}$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points.

Solution:

We first calculate the points in the plane corresponding to $t = 2$ and $t = 3$.

$$(x(2), y(2)) = (11, 15),\ (x(3), y(3)) = (19, 80).$$

Next we calculate the required derivatives using the chain rule.

$$\begin{aligned} \left.\frac{\partial f}{\partial t}\right|_{t=2} &= \left.\frac{\partial f}{\partial x}\right|_{(x,y)=(11,15)} \left.\frac{\partial x}{\partial t}\right|_{t=2} + \left.\frac{\partial f}{\partial y}\right|_{(x,y)=(11,15)} \left.\frac{\partial y}{\partial t}\right|_{t=2} \\ &= f_x(11,15)\ (3+2t)\Big|_{t=2} + f_y(11,15)\ (4t^3)\Big|_{t=2} \\ &= (3)(7) + (11)(32) \\ &= 373. \end{aligned}$$

Similarly we have

$$\begin{aligned} \left.\frac{\partial f}{\partial t}\right|_{t=3} &= \left.\frac{\partial f}{\partial x}\right|_{(x,y)=(19,80)} \left.\frac{\partial x}{\partial t}\right|_{t=3} + \left.\frac{\partial f}{\partial y}\right|_{(x,y)=(19,80)} \left.\frac{\partial y}{\partial t}\right|_{t=3} \\ &= f_x(19,80)\ (3+2t)\Big|_{t=3} + f_y(19,80)\ (4t^3)\Big|_{t=3} \\ &= (-7)(9) + (13)(108) \\ &= 1341. \end{aligned}$$