



Bilkent University

Quiz # 7  
Math 102-Section 11  
5 May 2023, Friday, Moodle Quiz  
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**Solution Key**

**Q-1)** Let  $z = f(x, y)$  where  $f$  is a differentiable function. We restrict this function to the curve with parametric equations  $(1 + 3t + t^2, t^4 - 1)$ ,  $t \in \mathbb{R}$ .

We also know that:

$$\begin{aligned} f_x(15, 11) &= 1, & f_x(11, 15) &= 3, & f_y(15, 11) &= 7 \\ f_y(11, 15) &= 11, & f_y(19, 80) &= 13, & f_y(80, 11) &= 17 \\ f_x(11, 80) &= 10, & f_x(80, 11) &= -1, & f_x(19, 80) &= -7 \\ f_y(11, 80) &= -6, & f_x(15, 80) &= 11, & f_y(15, 80) &= 16 \end{aligned}$$

Calculate  $\left. \frac{\partial f}{\partial t} \right|_{t=2}$  and  $\left. \frac{\partial f}{\partial t} \right|_{t=3}$ .

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points.

**Solution:**

We first calculate the points in the plane corresponding to  $t = 2$  and  $t = 3$ .

$$(x(2), y(2)) = (11, 15), \quad (x(3), y(3)) = (19, 80).$$

Next we calculate the required derivatives using the chain rule.

$$\begin{aligned} \left. \frac{\partial f}{\partial t} \right|_{t=2} &= \left. \frac{\partial f}{\partial x} \right|_{(x,y)=(11,15)} \left. \frac{\partial x}{\partial t} \right|_{t=2} + \left. \frac{\partial f}{\partial y} \right|_{(x,y)=(11,15)} \left. \frac{\partial y}{\partial t} \right|_{t=2} \\ &= f_x(11, 15) (3 + 2t) \Big|_{t=2} + f_y(11, 15) (4t^3) \Big|_{t=2} \\ &= (3)(7) + (11)(32) \\ &= 373. \end{aligned}$$

Similarly we have

$$\begin{aligned} \left. \frac{\partial f}{\partial t} \right|_{t=3} &= \left. \frac{\partial f}{\partial x} \right|_{(x,y)=(19,80)} \left. \frac{\partial x}{\partial t} \right|_{t=3} + \left. \frac{\partial f}{\partial y} \right|_{(x,y)=(19,80)} \left. \frac{\partial y}{\partial t} \right|_{t=3} \\ &= f_x(19, 80) (3 + 2t) \Big|_{t=3} + f_y(19, 80) (4t^3) \Big|_{t=3} \\ &= (-7)(9) + (13)(108) \\ &= 1341. \end{aligned}$$