Bilkent University

Quiz # 8
Math 102-Section 09
10 May 2023, Wednesday, Moodle Quiz
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Solution Key

Q-1) For the function $f(x, y) = 2x^3 - 3x^2 + y^3$, find its critical points and determine if these critical points are local minimum, local maximum or saddle points.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 10 points.

Solution:

For the critical points we use the first partial derivatives. We then solve

$$f_x(x, y) = 6x^2 - 6x = 0, \text{ and } f_y(x, y) = 3y^2 = 0.$$

Thus the critical points are $(0, 0)$ and $(1, 0)$.

Next we calculate the second partial derivatives to prepare for the second derivative test.

$$f_{xx}(x, y) = 12x - 6,\ f_{yy}(x, y) = 6y,\ f_{xy} = 0,\ \Delta(x, y) = f_{xx}f_{yy} - f_{xy}^2 = 72xy - 36y.$$

We see that $\Delta(0, 0) = 0$ and $\Delta(1, 0) = 0$, so the second derivative test does not apply.

However we notice that for any $\epsilon > 0$ we have

$$f(0, -\epsilon) < f(0, 0) < f(0, \epsilon),$$
$$f(1, -\epsilon) < f(1, 0) < f(1, \epsilon).$$

Now recall the definition of a saddle point: a critical point p for f is a saddle point if on every open disc U around p there are points $p_1, p_2 \in U$ with $f(p_1) < f(p) < f(p_2)$.

Hence both of our critical points are saddle points.

Here is a graph of the surface $z = f(x, y)$ around the critical points.