



Quiz # 10
 Math 102 Section 08 Calculus II
 29 April 2024 Monday
 Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

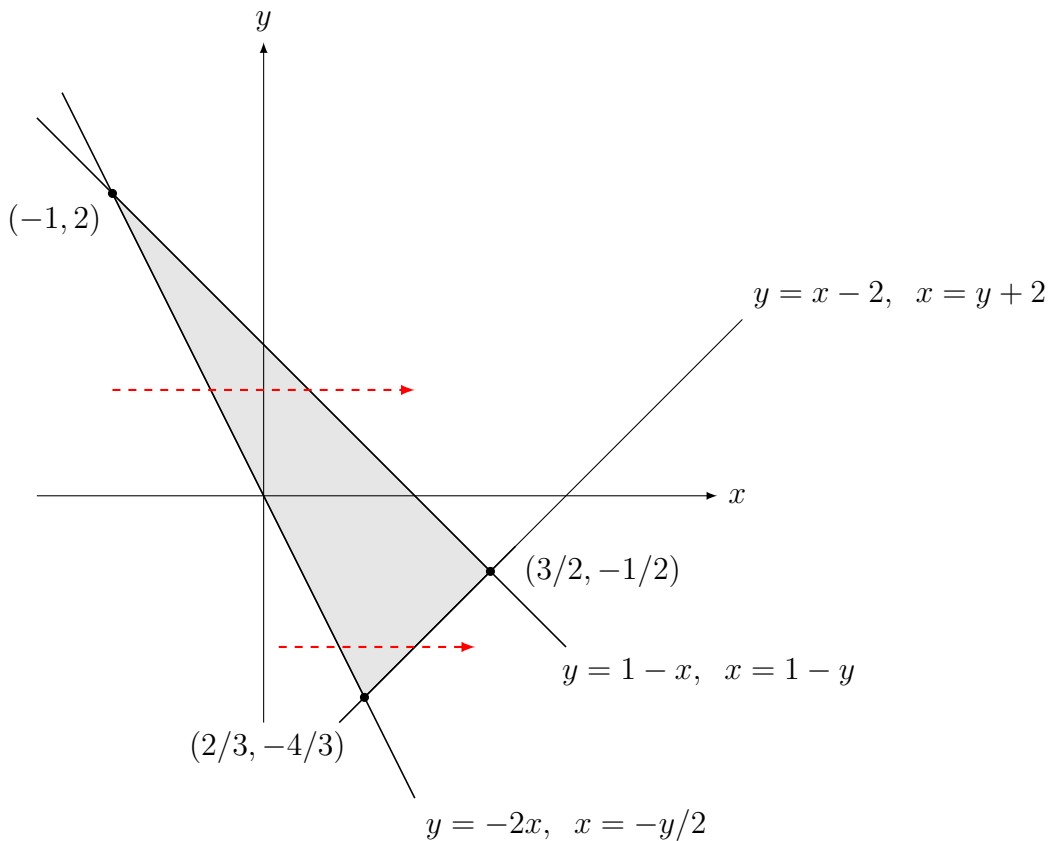
Q-1) Fill in the boxes below. No questions asked!

$$\int_{-1}^{2/3} \int_{-2x}^{1-x} dy dx + \int_{2/3}^{3/2} \int_{x-2}^{1-x} dy dx = \int_{\boxed{-4/3}}^{\boxed{-1/2}} \int_{\boxed{-y/2}}^{\boxed{y+2}} dx dy + \int_{\boxed{-1/2}}^{\boxed{2}} \int_{\boxed{-y/2}}^{\boxed{1-y}} dx dy = \boxed{\frac{25}{12}}$$

Grading: Each correctly filled box is 1 point, except the last box on the right which is 2 points.

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Solution:



Evaluation of the integrals are on the next page.

$$\int_{-1}^{2/3} \int_{-2x}^{1-x} dy dx = \int_{-1}^{2/3} \left(y \Big|_{-2x}^{1-x} \right) dx = \int_{-1}^{2/3} (1+x) dx = \left(x + \frac{x^2}{2} \Big|_{-1}^{2/3} \right) = \frac{25}{18}$$

$$\int_{2/3}^{3/2} \int_{x-2}^{1-x} dy dx = \int_{2/3}^{3/2} \left(y \Big|_{x-2}^{1-x} \right) dx = \int_{2/3}^{3/2} (3-2x) dx = \left(3x - x^2 \Big|_{2/3}^{3/2} \right) = \frac{25}{36}$$

Hence

$$\int_{-1}^{2/3} \int_{-2x}^{1-x} dy dx + \int_{2/3}^{3/2} \int_{x-2}^{1-x} dy dx = \frac{25}{18} + \frac{25}{36} = \frac{25}{12}.$$

Similarly,

$$\int_{-4/3}^{-1/2} \int_{-y/2}^{y+2} dx dy = \int_{-4/3}^{-1/2} \left(x \Big|_{-y/2}^{y+2} \right) dy = \int_{-4/3}^{-1/2} \left(\frac{3y}{2} + 2 \right) dy = \left(\frac{3y^2}{4} + 2y^2 \Big|_{-4/3}^{-1/2} \right) = \frac{25}{48}$$

$$\int_{-1/2}^2 \int_{-y/2}^{1-y} dx dy = \int_{-1/2}^2 \left(x \Big|_{-y/2}^{1-y} \right) dy = \int_{-1/2}^2 \left(1 - \frac{y}{2} \right) dy = \left(y - \frac{y^2}{4} \Big|_{-1/2}^2 \right) = \frac{25}{16}$$

Hence

$$\int_{-4/3}^{-1/2} \int_{-y/2}^{y+2} dx dy + \int_{-1/2}^2 \int_{-y/2}^{1-y} dx dy = \frac{25}{48} + \frac{25}{16} = \frac{25}{12}.$$