Oct 8, 2003

MATH 113 HOMEWORK 1

Turn in by Oct. 13, 2003 Monday.

1. Show by induction that $133 \mid 11^{n+2} + 12^{2n+1}$ for all integers $n \geq 0$.

2. Definition. If a and b are two unequal positive integers we define $\max(a,b)$ to be the larger of a and b. If $a = b$, we set $\max(a,b) = a = b$ (Thus $\max(3,5) = \max(5,3) = 5$, and $\max(4,4) = 4$).

Let A_n be the following statement: If a and b are any two positive integers such that $\max(a,b) = n$, then $a = b$.

Consider the following:
Theorem. A_n is true for every integer $n \geq 1$.
Proof. a) Show A_1 is true. Let a and b be two positive integers such that $\max(a,b) = 1$. Then $a \leq 1$ and $b \leq 1$. Since they are also positive integers we must have $a = b = 1$.
b) Assume that the statement is true for some positive integer $k \geq 1$, and prove it for $k+1$. Let a and b be any two positive integers such that $\max(a,b) = k+1$. Let $\alpha = a-1$, $\beta = b-1$. Then $\max(\alpha, \beta) = k$. Since the statement A_k is true, we have that $\alpha = \beta$, so $a = b$.
Corollary. $3 = 5$.
Proof. $\max(3,5) = 5$. Since A_5 is true, we have that $3 = 5$.

Question. What is the fallacy in the above proof?

3. Show that for all integers $n \geq 3$,

$$1^3 + 3^3 + 5^3 + \cdots + (2n+1)^3 = 18n^3 + 4n^2 - 8n - 2.$$