

Due Date: February 22, 2012 Wednesday

NAME:.....

Ali Sinan Sertöz

STUDENT NO:.....

Math 114 Calculus – Homework 1

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are 4 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Q-1)

i. Let $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n$. Find, if it exists, $\lim_{n \rightarrow \infty} a_n$.

ii. Let $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2} - \cdots - \frac{1}{n^2}$. Find, if it exists, $\lim_{n \rightarrow \infty} a_n$.

Solution:

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Q-2) Let $\alpha \geq 0$ be any non-negative real number. Define a sequence a_n recursively as follows.

$$a_1 = \alpha, \quad a_{n+1} = \sqrt{1 + 2a_n} \quad \text{for } n \geq 1.$$

For which values of α does the sequence a_n converge? Find $\lim_{n \rightarrow \infty} a_n$ when it exists.

Solution:

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Q-3)

i. For which values of $\alpha \in \mathbb{R}$ does the series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^\alpha}$ converge?

ii. For which values of $\alpha \in \mathbb{R}$ does the series $\sum_{n=3}^{\infty} \frac{1}{n(\ln \ln n)^\alpha}$ converge?

Solution:

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Q-4)

i. Find $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$. Let $a_n = \frac{n!}{n^n}$. Does the series $\sum_{n=1}^{\infty} a_n$ converge?

ii. Does the series $\sum_{n=1}^{\infty} \frac{1}{1 + \frac{1}{2} + \dots + \frac{1}{n}}$ converge?

Solution: