Date: 28 May 2012, Monday

Time: 10:00-12:00

Ali Sinan Sertöz

NAME:..

STUDENT NO:..

Math 114 Calculus II – Make-up Exam – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

PLEASE READ:

Check that there are 5 questions on your exam booklet. Write your name on the top of every page. Show your work in reasonable detail, if that is necessary. A correct answer without proper reasoning may not get any credit, except in the fill-in-the-blanks type problems.

NAME: STUDENT NO:

Q-1) Write your answers on the spaces provided. No partial credits.

$f(x) = \sin(e^{\cos x})$	$f'(x) = \cos(e^{\cos x})e^{\cos x}(-\sin x)$
$f(x) = (\tan x)^{\sec x}$	$f'(x) = (\tan x)^{\sec x}[\sec x \tan x \ln \tan x + \dfrac{\sec^3 x}{\tan x}]$
$f(x) = \pi^x + \pi^\pi + x^\pi$	$f'(x) = \pi^x \ln \pi + \pi x^{\pi-1}$
$f(x) = \arcsin(e^x)$	$f'(x) = \dfrac{e^x}{\sqrt{1-e^{2x}}}$
$f(x,y) = y^7 + x^5y^6 + x + x^3$, $x(t) = \cos t + \sin t + t + 1$, $y(t) = e^t + \ln(1+t^2) + t$, $h(t) = f(x(t), y(t))$.	$h'(0) = 584$

Nothing below this line will be read on this page!

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Q-2 Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n.$$

Find also the convergence behavior of the series at the end points of its interval of convergence.
Hint: $n^{1/3} < \frac{e^n n!}{n^n} < 3n,$ *for all* $n = 1, 2, 3, \ldots.$

Solution:

Let $b_n(x) = \dfrac{n^n}{n!} x^n$. Apply ratio test to find

$$\left| \frac{b_{n+1}(x)}{b_n(x)} \right| = (1 + 1/n)^n |x| \to e|x| \text{ as } n \to \infty.$$

The series absolutely converges for $|x| < 1/e$, hence the radius of convergence is $1/e$.

From the hint we know that $b_n(1/e) > 1/(3n)$, hence $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n! e^n}$ diverges by comparison with the harmonic series.

From the hint we also know that $b_n(1/e) < 1/(n^{1/3})$. Check also that

$$b_{n+1}(1/e) = \frac{(1 + 1/n)^n}{e} b_n(1/e) < b_n(1/e).$$

Hence $b_n(1/e) \downarrow 0$ as $n \to \infty$. Therefore, by the alternating series test, the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n! e^n}$ converges.

Finally, we conclude that the interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$ is $[-1/e, 1/e)$.

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Q-3) The point $(-1, 0)$ is a critical point for the function $f(x, y) = x^3 + xy^2 + y^2 - 3x$. What is the nature of this critical point? Is it a local/global min/max or a saddle point?

Solution:

The second derivative test fails here so we have to examine the behavior of the function at $(-1, 0)$. We have

$$f(-1 + s, t) - f(-1, 0) = -3s^2 + s^3 + st^2.$$

We notice that

$$f(-1 + s, 0) - f(-1, 0) = s^2(3 - s) < 0 \text{ for all } |S| < 3$$

and

$$f(-1 + \frac{t^2}{3}, t) - f(-1, 0) = \frac{t^6}{27} > 0 \text{ for all } |t|.0.$$

This shows that in every neighborhood of $(-1, 0)$, there are points where the sign of $f(x, yt) - f(-1, 0)$ becomes both positive and negative. Hence this is a saddle point.

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Q-4) Evaluate the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x+y}{x^2+y^2}\, dydx.$$

Solution:

$$\begin{aligned}
\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x+y}{x^2+y^2}\, dydx &= \int_0^{\pi/2} \int_0^{2\cos\theta} (\cos\theta + \sin\theta)\, drd\theta \\
&= \int_0^{\pi/2} (2\cos^2\theta + 2\sin\theta\cos\theta)\, d\theta \\
&= \int_0^{\pi/2} (1 + \cos 2\theta + \sin 2\theta)\, d\theta \\
&= \left(\theta + \frac{1}{2}\sin 2\theta - \frac{1}{2}\cos 2\theta \Big|_0^{\pi/2} \right) \\
&= \frac{\pi}{2} + 1.
\end{aligned}$$

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Q-5) Let D be the region lying in upper half space $z \geq 0$, bounded by the solid cone $3z^2 \geq x^2 + y^2$ and the ball $x^2 + y^2 + z^2 \leq 1$. Write the limits of integration for the following iterated integrals for Cartesian, spherical and cylindrical coordinates respectively. Evaluate one of these integrals and write the answer in the box provided. No partial credits.

(Grading: each box=1 point, except the answer box which is 2 credits.)

$$\iiint_D dV = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} dz\, dx\, dy$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} r\, dz\, dr\, d\theta$$

$$= \boxed{}\,.$$

Solution:

$$\iiint_D (6+4y)\, dV = \int_{\boxed{-\sqrt{3}/2}}^{\boxed{\sqrt{3}/2}} \int_{\boxed{-\sqrt{3-4x^2}/2}}^{\boxed{\sqrt{3-4x^2}/2}} \int_{\boxed{\sqrt{(x^2+y^2)/3}}}^{\boxed{\sqrt{1-x^2-y^2}}} dz\, dx\, dy$$

$$= \int_{\boxed{0}}^{\boxed{2\pi}} \int_{\boxed{0}}^{\boxed{\pi/3}} \int_{\boxed{0}}^{\boxed{1}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_{\boxed{0}}^{\boxed{2\pi}} \int_{\boxed{0}}^{\boxed{\sqrt{3}/2}} \int_{\boxed{r/\sqrt{3}}}^{\boxed{\sqrt{1-r^2}}} r\, dz\, dr\, d\theta$$

$$= \boxed{\pi/3}\,.$$