

Summer 2007-08 MATH 116 Homework 1

Due on June 18.

No late homework will be accepted.

1. Let $f(x, y) = \frac{\sqrt{xy}}{\ln(x^2 + y^2 - 4)}$.
- (a) Find the domain of the function f .
- (b) Find the range of the function f .

2. Evaluate the limit

$$\lim_{(x,y) \rightarrow (0,0)} \int_0^{x^2 y^2} \frac{\sin(t+1)}{(t+1)(x^2+y^2)} dt$$

3. (a) Let $f(t)$ be a differentiable function. If $u(x, y) = f(\frac{x}{y})$ for $y \neq 0$, prove that u satisfies the partial-differential equation

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0.$$

- (b) Find a solution to the partial-differential equation in part (a) such that $u(1, 1) = 2$ and $u_x(x, \frac{1}{x}) = \frac{1}{x}$ for all $x \neq 0$.

4. Let $f(x) = \int_{x^4+x^2}^{x^6} \sqrt{t^3+x^2} dt$. Then find $f'(x)$.

(Hint: See the solution of exercise 49 in Section 14.4)

5. Let $L(x, y)$ be the linearization of the function $f(x, y) = x^2 + y^2$ at $(1, 1)$. Let $E(x, y)$ be the error function defined by $E(x, y) = f(x, y) - L(x, y)$. In what direction does $E(x, y)$ increase most rapidly at $(2, 0)$ and what is the rate of change of $E(x, y)$ in this direction?