Date: July 17, 2009, Friday NAME:.....

STUDENT NO:.....

SECTION NUMBER: .....

Math 116 Calculus – QUIZ # 10 – Solutions

Question: Use the surface integral in Stokes' theorem to calculate the circulation of the field $\mathbf{F} = 2y\mathbf{i} + 3x\mathbf{j} + z^2\mathbf{k}$ around the circle $x^2 + y^2 = 9$ in the xy-plane, counterclockwise when viewed above.

Solution:

$$\nabla \times \mathbf{F} = \mathbf{k}$$

Surface S bounded by C is the disc $x^2 + y^2 \leq 9$ in the xy-plane. Hence $\mathbf{n} = \mathbf{k}$. Then

$$\text{Circulation} = \oint_C \mathbf{F} \cdot d\mathbf{r} = \int\int_R [\text{Curl}\mathbf{F}] \cdot \mathbf{n}\, d\sigma = \int\int_R d\sigma = 9\pi.$$

Question: Use the surface integral in Stokes' theorem to calculate the circulation of the field $\mathbf{F} = 3y\mathbf{i}+2x\mathbf{j}+(z^3+1)\mathbf{k}$ around the circle $x^2+y^2 = 4$ in the xy-plane, counterclockwise when viewed above.

Solution:

$$\nabla \times \mathbf{F} = -\mathbf{k}$$

Surface S bounded by C is the disc $x^2 + y^2 \leq 4$ in the xy-plane. Hence $\mathbf{n} = \mathbf{k}$. Then

$$\text{Circulation} = \oint_C \mathbf{F} \cdot d\mathbf{r} = \int\int_R [\text{Curl}\mathbf{F}] \cdot \mathbf{n}\, d\sigma = -\int\int_R d\sigma = -4\pi.$$