

Date: June 26, 2009, Friday

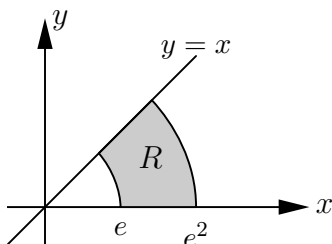
Math 116 Calculus – QUIZ # 4

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**Question 1A:** Integrate

$$f(x, y) = \frac{\ln(x^2 + y^2)}{x^2 + y^2}$$

over the region  $R$  given below



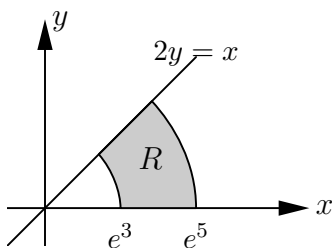
**Solution:** Use polar substitution.

$$\begin{aligned} \int \int_R \frac{\ln(x^2 + y^2)}{x^2 + y^2} &= 2 \int_0^{\pi/4} \int_e^{e^2} \frac{\ln r}{r} dr d\theta \\ &= \int_0^{\pi/4} \left( (\ln r)^2 \Big|_e^{e^2} \right) d\theta \\ &= 3 \int_0^{\pi/4} d\theta = \frac{3\pi}{4}. \end{aligned}$$

**Question 1B:** Integrate

$$f(x, y) = \frac{\ln(x^2 + y^2)}{x^2 + y^2}$$

over the region  $R$  given below



**Solution:** Use polar substitution. Let  $0 < \theta_0 < \pi/2$  be such that  $\tan \theta_0 = 1/2$ .

$$\begin{aligned} \int \int_R \frac{\ln(x^2 + y^2)}{x^2 + y^2} &= 2 \int_0^{\theta_0} \int_{e^3}^{e^5} \frac{\ln r}{r} dr d\theta \\ &= \int_0^{\theta_0} \left( (\ln r)^2 \Big|_{e^3}^{e^5} \right) d\theta \\ &= 16 \int_0^{\theta_0} d\theta = 16 \theta_0. \end{aligned}$$