
Math 116 Calculus – QUIZ # 5

Question: Evaluate the triple integral $\int \int \int_D z \, dV$ where D is the region between the planes $x + y + 2z = 4$ and $2x + 2y + z = 8$ in the first octant.

Solution:

$$\begin{aligned}
 \int \int \int_D z \, dV &= \int_0^4 \int_0^{4-y} \int_{2-x/2-y/2}^{8-2x-2y} z \, dz \, dx \, dy \\
 &= \int_0^4 \int_0^{4-y} \left[\frac{1}{2} (8 - 2x - 2y)^2 - \frac{1}{2} (2 - (1/2)x - (1/2)y)^2 \right] dx \, dy \\
 &= \int_0^4 \int_0^{4-y} \left[30 - 15x - 15y + \frac{15}{8}x^2 + \frac{15}{4}xy + \frac{15}{8}y^2 \right] dx \, dy \\
 &= \int_0^4 (40 - 30y + (15/2)y^2 - (5/8)y^3) \, dy \\
 &= 40.
 \end{aligned}$$

Question: Evaluate the triple integral $\int \int \int_D x \, dV$ where D is the region between the planes $z + y + 2x = 4$ and $2z + 2y + x = 8$ in the first octant.

Solution:

$$\begin{aligned}
 \int \int \int_D x \, dV &= \int_0^4 \int_0^{4-y} \int_{2-z/2-y/2}^{8-2z-2y} x \, dx \, dz \, dy \\
 &= \int_0^4 \int_0^{4-y} \left[\frac{1}{2} (8 - 2z - 2y)^2 - \frac{1}{2} (2 - (1/2)z - (1/2)y)^2 \right] dz \, dy \\
 &= \int_0^4 \int_0^{4-y} \left[30 - 15z - 15y + \frac{15}{8}z^2 + \frac{15}{4}zy + \frac{15}{8}y^2 \right] dz \, dy \\
 &= \int_0^4 (40 - 30y + (15/2)y^2 - (5/8)y^3) \, dy \\
 &= 40.
 \end{aligned}$$