Date: July 8, 2009, Wednesday NAME:.....

STUDENT NO:.....

SECTION NUMBER: .....

Math 116 Calculus – QUIZ # 7

Question: Find the (counterclockwise) circulation of the vector field $\mathbf{F} = (x-y)\mathbf{i} + x\mathbf{j}$ around the circle $x^2 + y^2 = 1$.

Solution: Let C be the positively oriented circle $x^2+y^2 = 1$. Use the usual parametrization $\mathbf{r} = (\cos t, \sin t)$, $0 \le t \le 2\pi$. Observe that $(dx, dy) = (-y, x)\, dt$ on C.

$$\begin{aligned}
\text{Circulation of } \mathbf{F} &= \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C (M dx + N dy) = \int_0^{2\pi} (Nx - My)\, dt \\
&= \int_0^{2\pi} \left(x^2 + (x-y)(-y)\right)\, dt \\
&= \int_0^{2\pi} (1 - \sin t \cos t)\, dt = 2\pi.
\end{aligned}$$

Question: Find the flux of the vector field $\mathbf{F} = y\mathbf{i} + (x-y)\mathbf{j}$ across the circle $x^2 + y^2 = 1$.

Solution: Using the above set up, we have

$$\begin{aligned}
\text{Flux of } \mathbf{F} &= \oint_C \mathbf{F}\cdot \mathbf{n}\, ds = \oint_C (M dy - N dx) = \int_0^{2\pi} (Mx + Ny)\, dt \\
&= \int_0^{2\pi} \left(2xy - y^2\right)\, dt \\
&= \int_0^{2\pi} (2\cos t \sin t - \sin^2 t)\, dt = -\pi.
\end{aligned}$$