Due Date: 21 December 2015, Monday Time: Class Time

NAME:

STUDENT NO: $\qquad$

## Math 202 Complex Analysis - Homework 5

| 1 | 2 | 3 | 4 | 5 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 20 | 20 | 20 | 20 | 20 | 100 |

Please do not write anything inside the above boxes!
Check that there are 5 questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

## Rules for Homework and Take-Home Exams

(1) You may discuss the problems only with your classmates or with me. In particular you may not ask your assigned questions or any related question to online forums.
(2) You may use any written source be it printed or online. Google search is perfectly acceptable.
(3) It is absolutely mandatory that you write your answers alone. Any similarity with your written words and any other solution or any other source that I happen to know is a direct violation of honesty.
(4) You must obey the usual rues of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends.
(5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME:
Q-1) Evaluate the integral $\int_{0}^{\infty} \frac{x^{2}}{x^{4}+6 x^{2}+13} d x$.

## Solution:

Q-2) Let $f(z)=\frac{z^{2}}{\left(z^{4}+1\right)^{2}}$.
(a) Find all poles of $f$.
(b) Show in detail how you calculate the residue of $f$ at one of the poles.
(c) Write all the residues of $f$ at all the poles. Do not show your work.

## Solution:

Q-3) Evaluate the integral $\int_{0}^{\infty} \frac{x^{2}}{\left(x^{4}+1\right)^{2}} d x$.

## Solution:

Q-4) Evaluate the integral $\int_{0}^{\infty} \frac{x \sin x}{x^{4}+1} d x$.
Solution:

Q-5) For any positive integer $n$, define the function

$$
f_{n}(z)=\frac{1}{z} \prod_{k=1}^{n} \frac{z}{1-k z}
$$

Let

$$
T_{n}=\sum_{k=1}^{n} \operatorname{Res}_{z=1 / k} f(z)
$$

Show that

$$
T_{n}=\frac{(-1)^{n}}{n!}
$$

## Solution:

