Math 206 Complex Calculus
Quiz-1
Solutions

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1) Find all the fourth roots of **(a)** $z_0 = -8 + 8\sqrt{3}i$, **(b)** $z_0 = -8 - 8\sqrt{3}i$.

Solution: Let $\alpha_0 = \mathrm{Arg}(z_0)$, where $0 \leq \alpha_0 < 2\pi$. It is determined by plotting z_0 on the xy-plane. Now let $\theta_k = \dfrac{\alpha_0 + 2k\pi}{4}$ for $k = 0, 1, 2, 3$. Note that in both cases $|z_0| = 16 = 2^4$, and $c_k = 2(\cos\theta_k + i\sin\theta_k)$ for $k = 0, 1, 2, 3$ are all the required fourth roots of z_0.

Solution a: $\alpha_0 = \dfrac{2\pi}{3}$.
$\theta_k = \left(\dfrac{2\pi}{3} + 2k\pi\right)\dfrac{1}{4}$, so: $\theta_0 = \dfrac{\pi}{6}$, $\theta_1 = \dfrac{4\pi}{6} = \pi - \dfrac{\pi}{3}$, $\theta_2 = \dfrac{7\pi}{6} = \pi + \dfrac{\pi}{6}$,
$\theta_3 = \dfrac{10\pi}{6} = 2\pi - \dfrac{\pi}{3}$.

Now a straightforward calculation gives

$c_0 = \sqrt{3} + i$, $c_1 = -1 + i\sqrt{3}$, $c_2 = -\sqrt{3} - i$, $c_3 = 1 - i\sqrt{3}$.

Solution b: $\alpha_0 = \dfrac{4\pi}{3}$.
$\theta_k = \left(\dfrac{4\pi}{3} + 2k\pi\right)\dfrac{1}{4}$, so: $\theta_0 = \dfrac{\pi}{3}$, $\theta_1 = \dfrac{5\pi}{6} = \pi - \dfrac{\pi}{6}$, $\theta_2 = \dfrac{8\pi}{6} = \pi + \dfrac{\pi}{3}$,
$\theta_3 = \dfrac{11\pi}{6} = 2\pi - \dfrac{\pi}{6}$.

Now a straightforward calculation gives

$c_0 = 1 + i\sqrt{3}$, $c_1 = -\sqrt{3} + i$, $c_2 = -1 - i\sqrt{3}$, $c_3 = \sqrt{3} - i$.